Proving Weakly Consistent Applications Correct

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Joint work with
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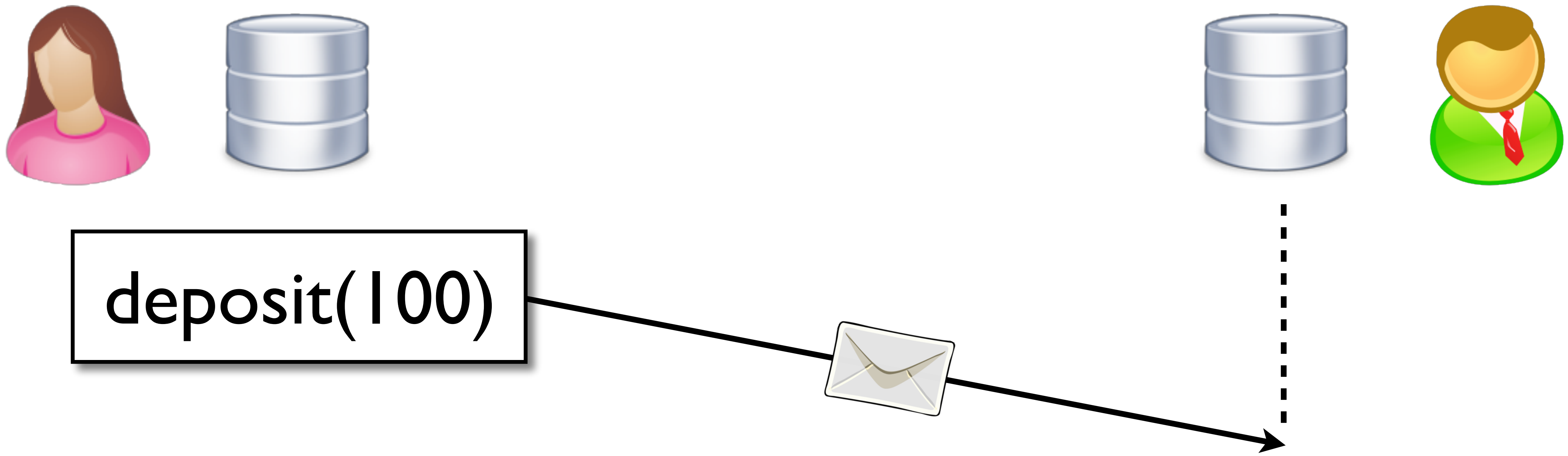
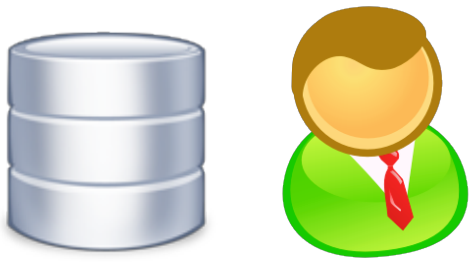
Eventually consistent databases



- No synchronisation: process an update locally, propagate effects to other replicas later
- Weakens consistency: deposit seen with a delay

balance $\geq$ 0



balance = 100

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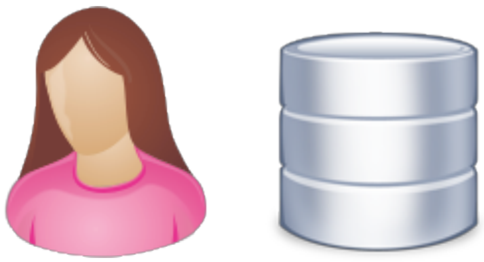
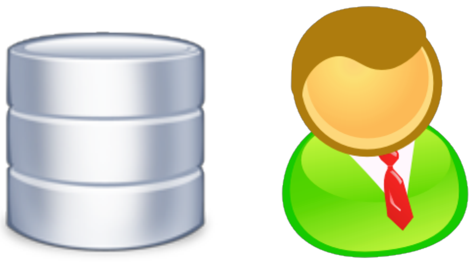


balance ≥ 0



balance = 100

withdraw(100) : ✔

balance = 0

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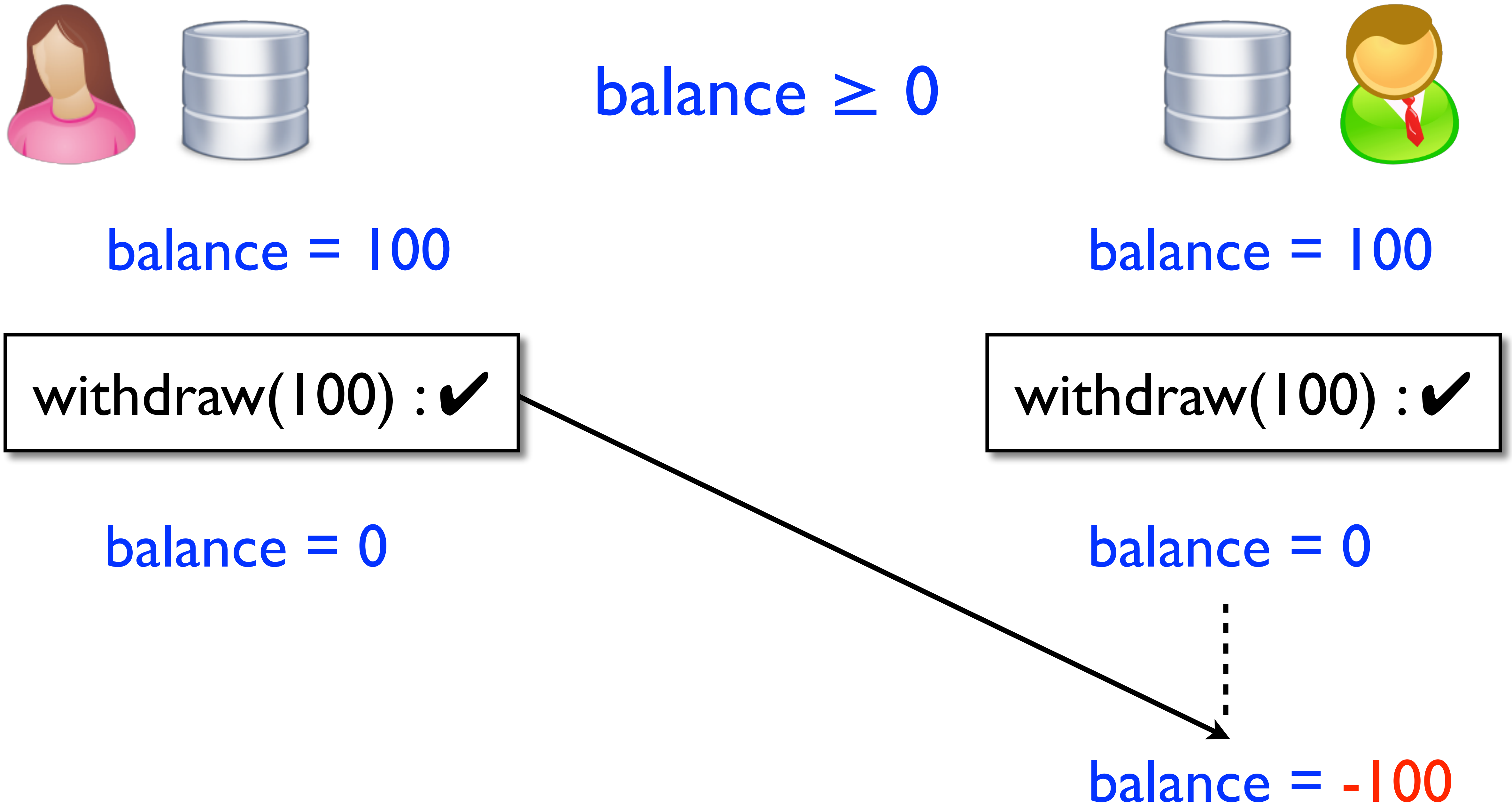

balance ≥ 0

balance = 100

withdraw(100) : ✔

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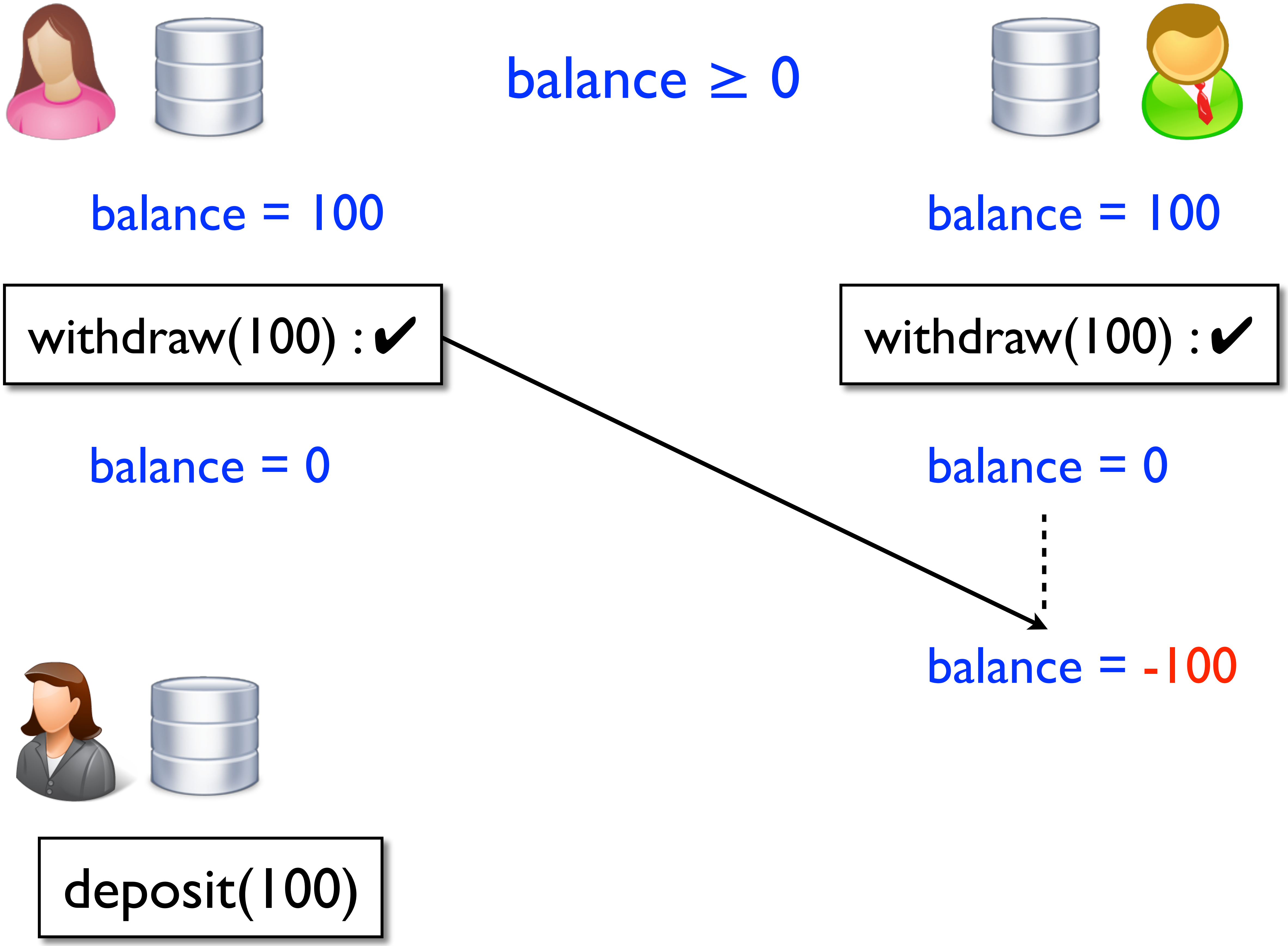
balance = 100

withdraw(100) : ✔

balance = 0

balance = -100

balance ≥ 0



balance = 100

withdraw(100) : ✔

balance = 0

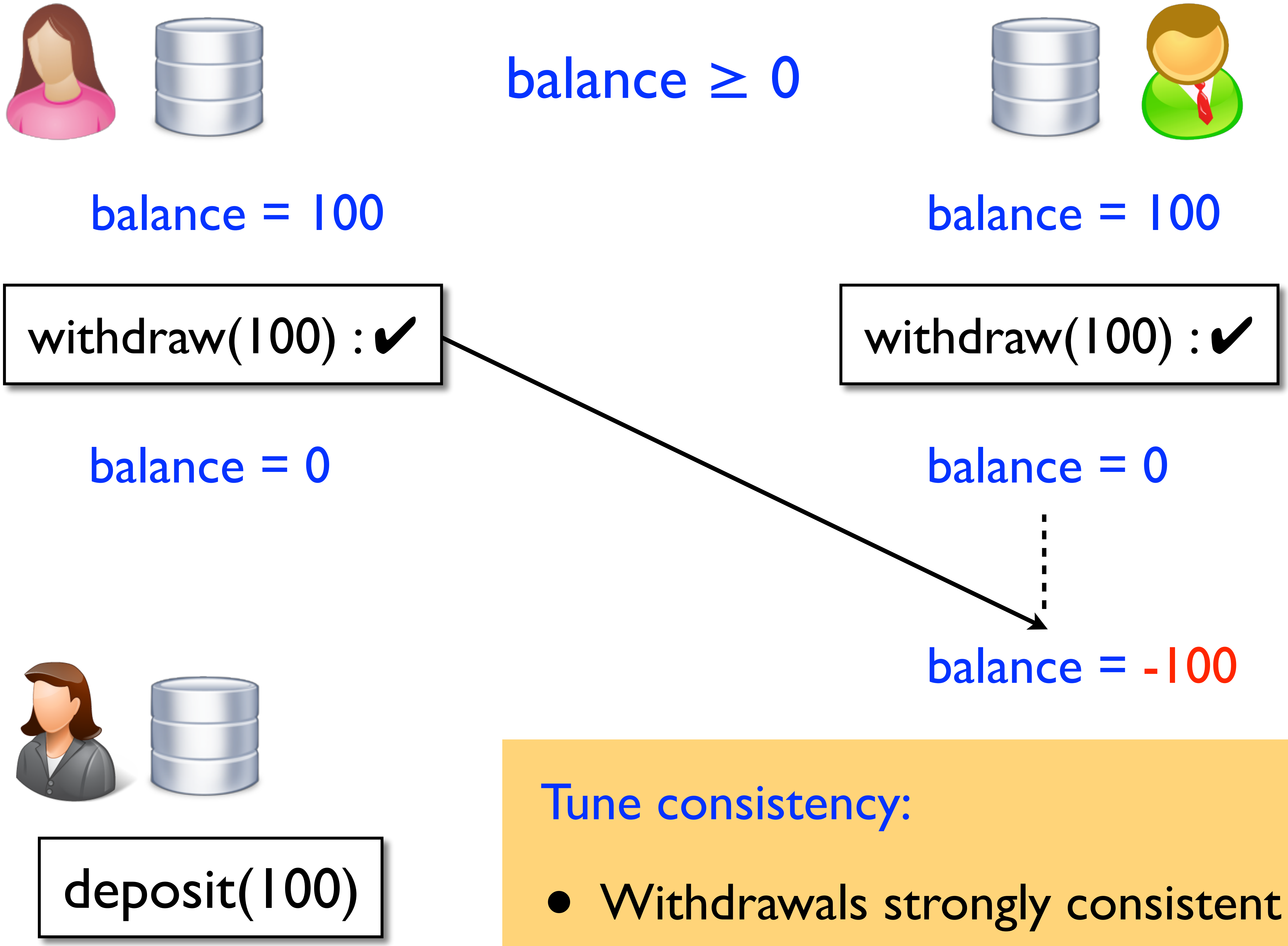
deposit(100)

balance = 100

withdraw(100) : ✔

balance = 0

balance = -100



balance ≥ 0

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balance = 0

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deposit(100)

Tune consistency:

- Withdrawals strongly consistent
- Deposits eventually consistent

Consistency choices

- Databases with multiple consistency levels:
  - Commercial: Amazon DynamoDB, Basho Riak, Microsoft DocumentDB
  - Research: Li+ OSDI'12; Terry+ SOSP'13; Balegas+ EuroSys'15...
- Pay for stronger semantics with latency, possible unavailability and money
- Hard to figure out the minimum consistency necessary to maintain correctness - proof rule and tool

Consistency model

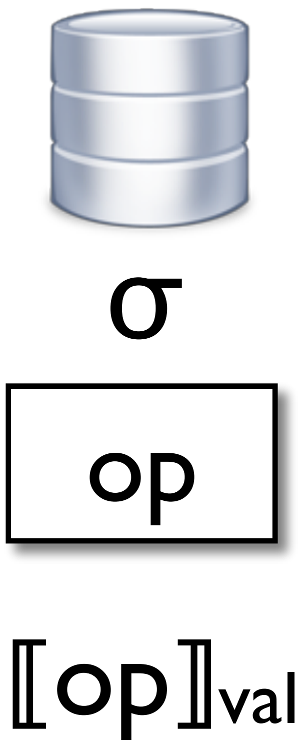
- Generic model - not implemented, but can encode many existing models that are:

  RedBlue consistency [Li+ 2012],
  reservation locks [Balegas+ 2015],
  parallel snapshot isolation [Sovran+ 2011], ...

- Causal consistency as a baseline: observe an update ➜ observe the updates it depends on

- A construct for strengthening consistency on demand

Operation semantics



σ

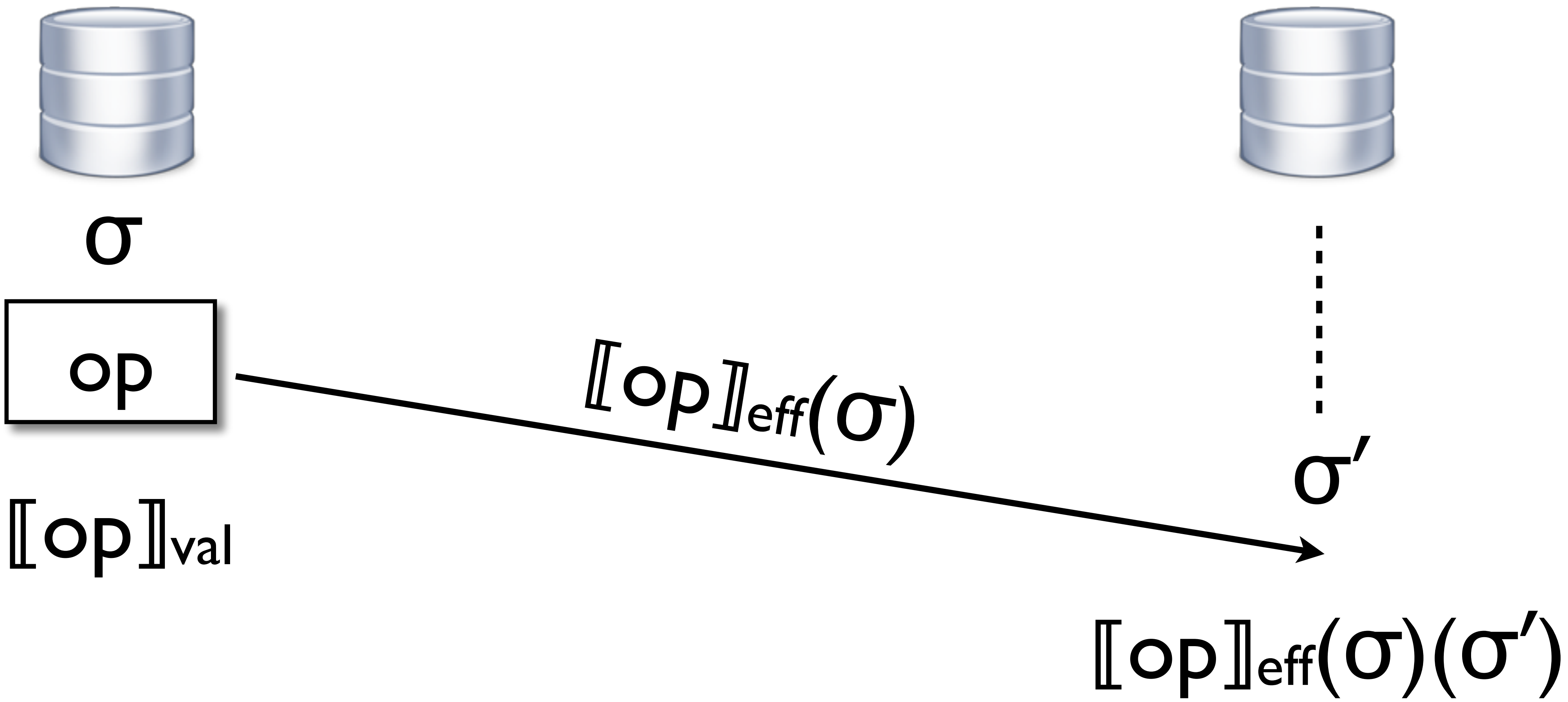
op

$[\![op]\!]_{val}$

Replica states: $\sigma \in State$

Return value: $[\![op]\!]_{val} \in State \rightarrow Value$

Operation semantics



σ

op

$[\![op]\!]_{eff}(\sigma)$

σ'

$[\![op]\!]_{val}$

$[\![op]\!]_{eff}(\sigma)(\sigma')$

Replica states: $\sigma \in State$

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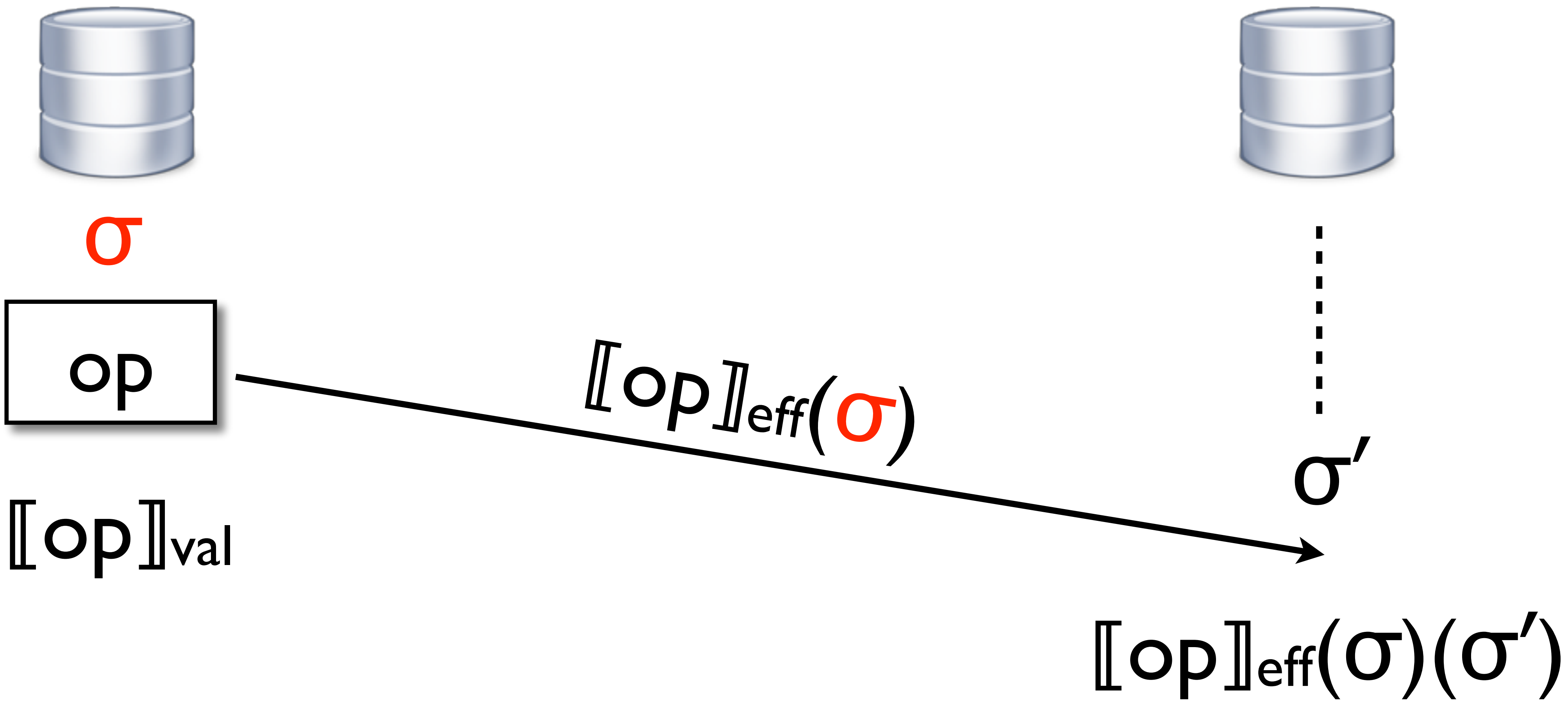
Effector: $[\![op]\!]_{eff} \in State \rightarrow (State \rightarrow State)$

Operation semantics



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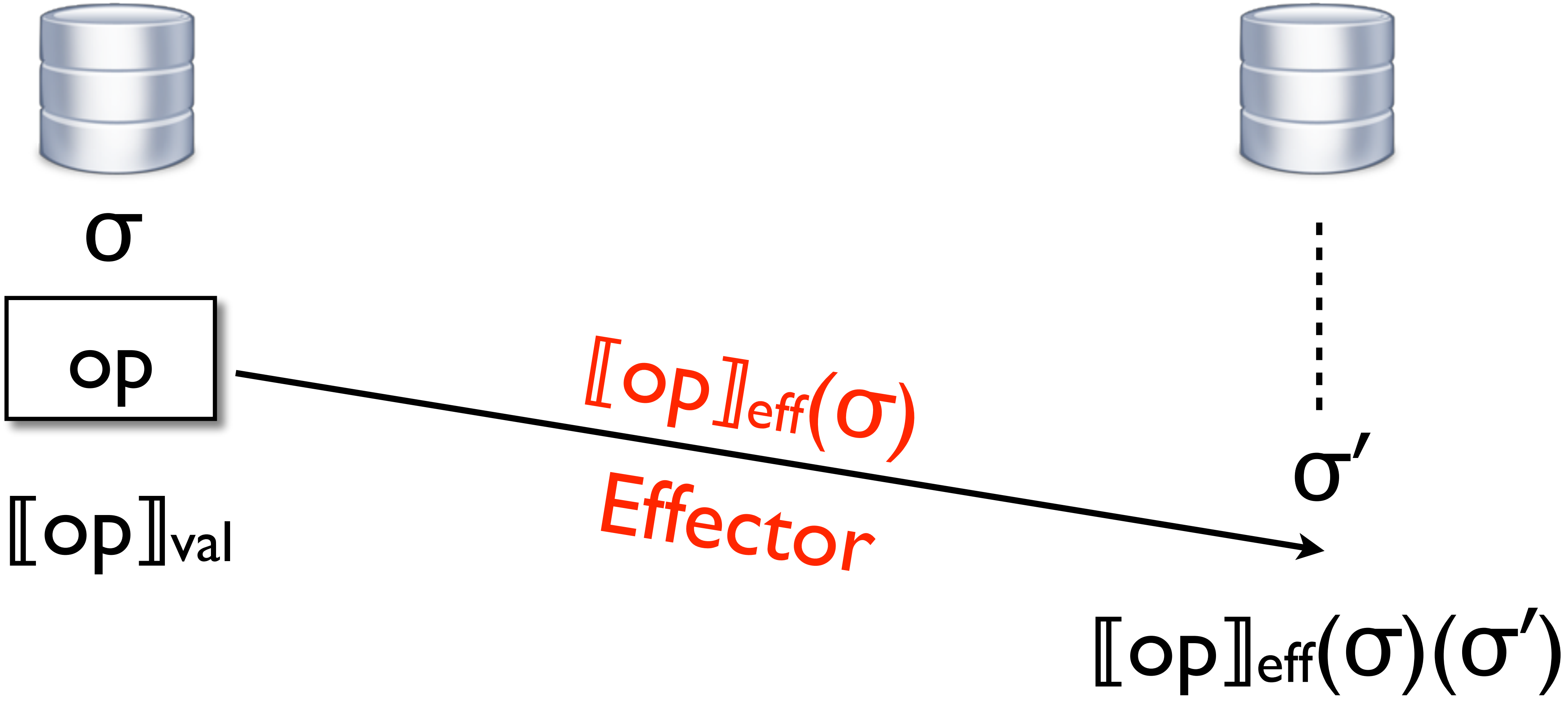

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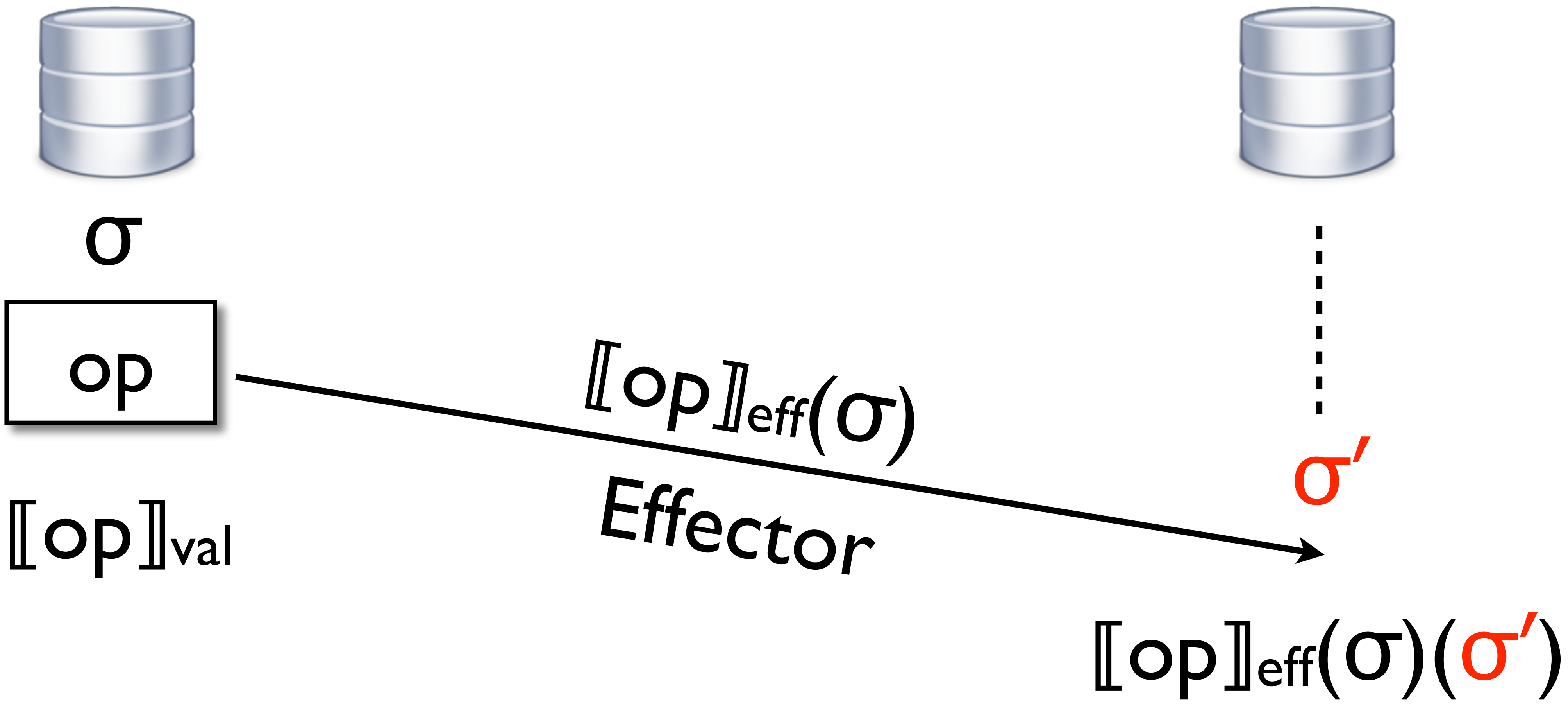

σ

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$\llbracket op \rrbracket_{eff}(\sigma)$

Effector

σ'

$\llbracket op \rrbracket_{val}$

$\llbracket op \rrbracket_{eff}(\sigma)(\sigma')$

Replica states: $\sigma \in \mathrm{State}$

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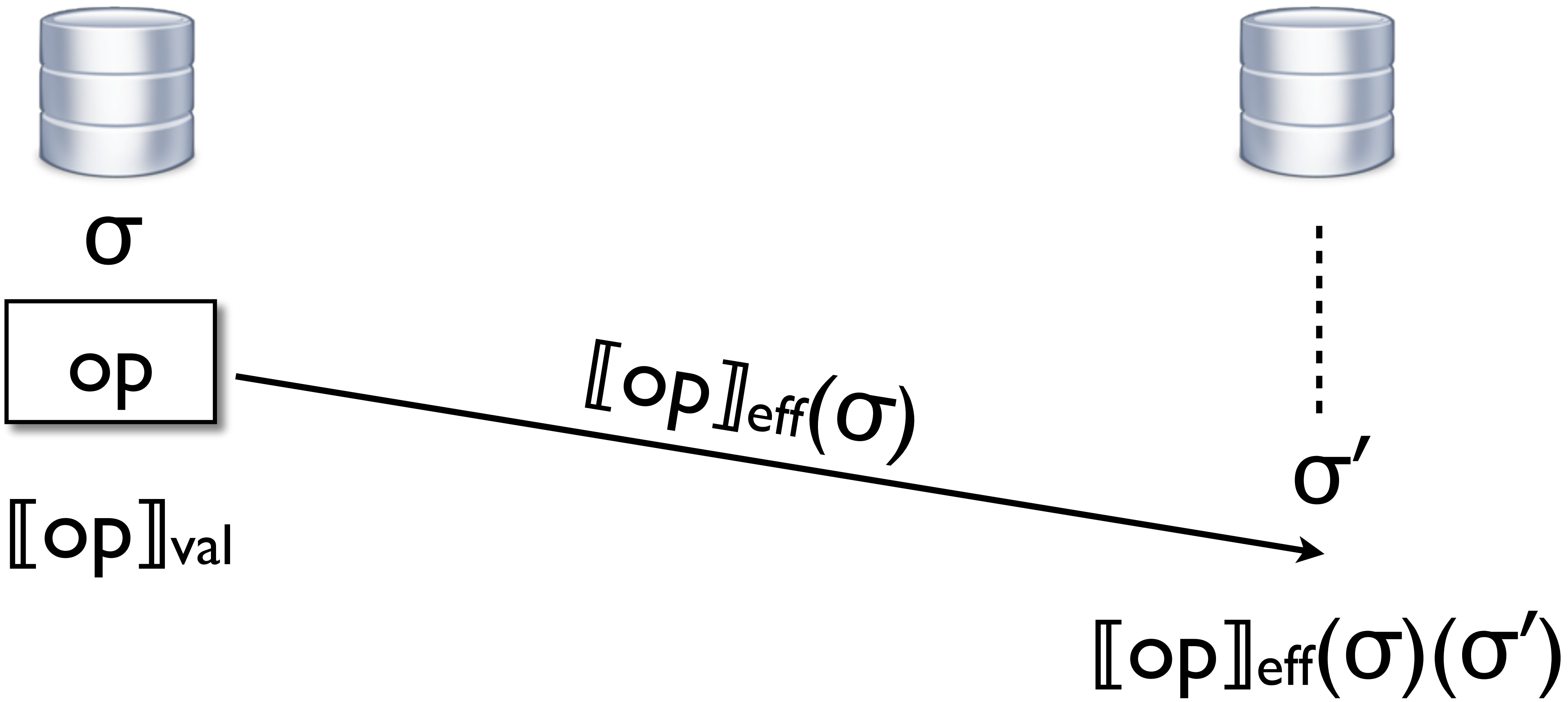


σ

op

$[\![op]\!]_{val}$

$[\![op]\!]_{eff}(\sigma)$

σ'

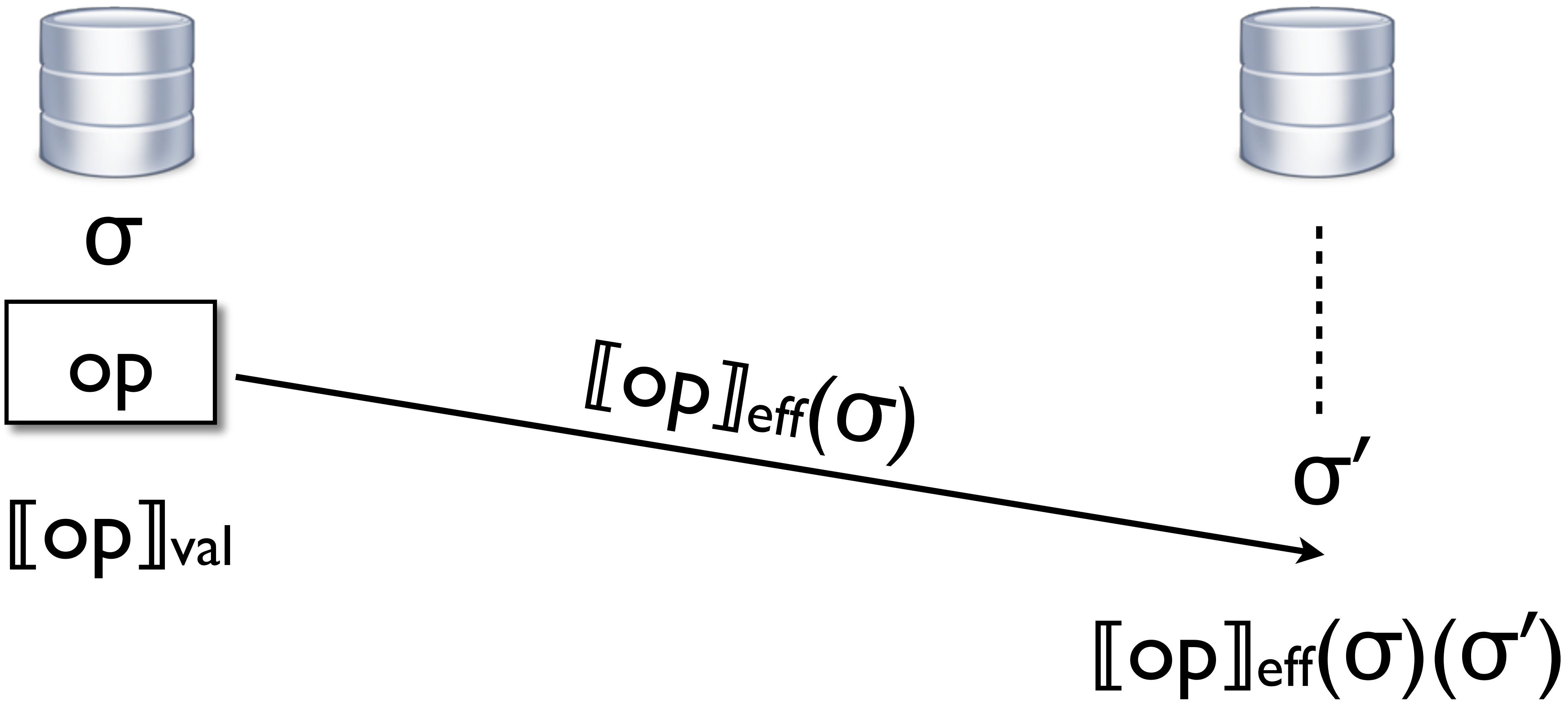
$[\![op]\!]_{eff}(\sigma)(\sigma')$

$\text{State} \;=\; Z$

$[\![balance()]\!]_{val}(\sigma) \;=\; \sigma$

$[\![balance()]\!]_{eff}(\sigma) \;=\; \lambda\sigma.\,\sigma$

Operation semantics



σ

op

$[\![\mathrm{op}]\!]_{\mathrm{val}}$

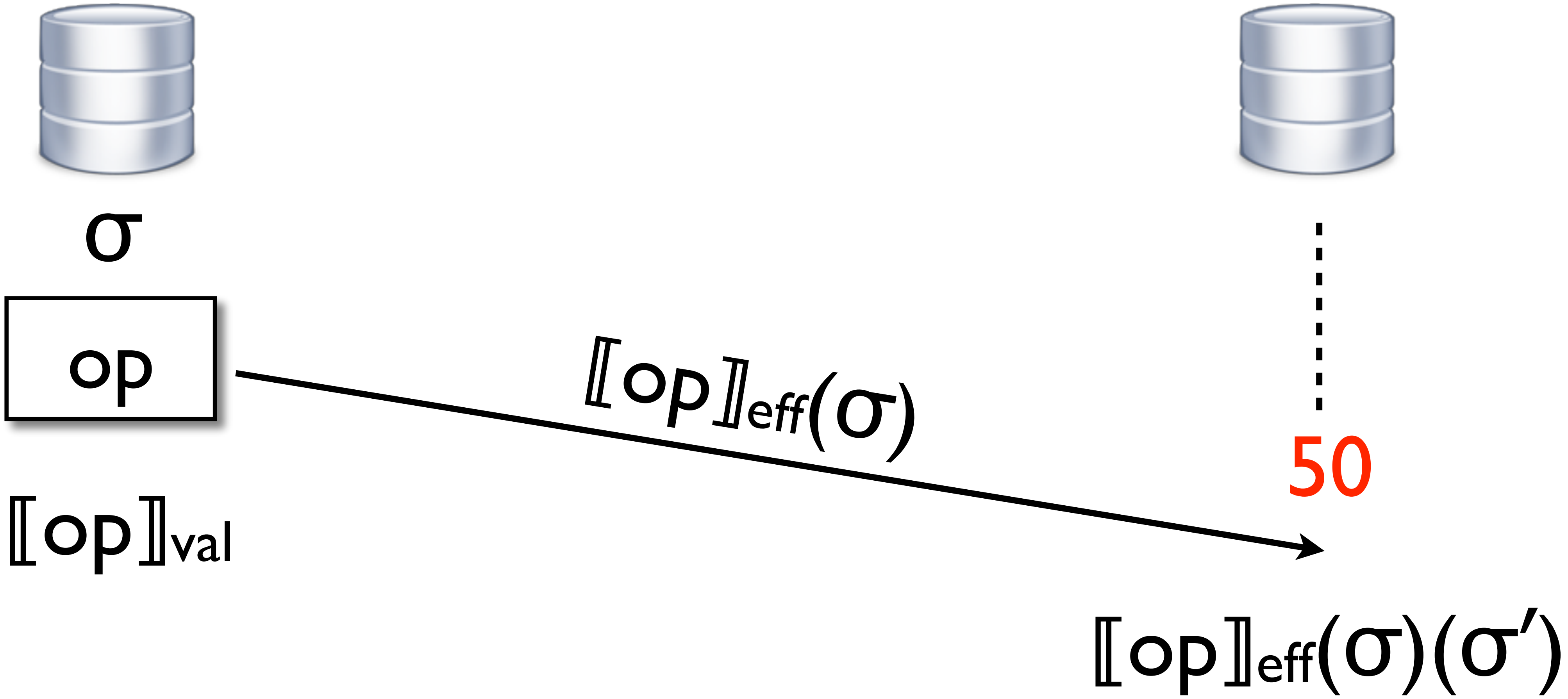
$[\![\mathrm{op}]\!]_{\mathrm{eff}}(\sigma)$

σ'

$[\![\mathrm{op}]\!]_{\mathrm{eff}}(\sigma)(\sigma')$

$$[\![\mathrm{deposit}(100)]\!]_{\mathrm{eff}}(\sigma) \;=\; \lambda\sigma'.\,(\sigma' + 100)$$

Operation semantics



σ

op

$[\![\mathrm{op}]\!]_{\mathrm{val}}$

$[\![\mathrm{op}]\!]_{\mathrm{eff}}(\sigma)$

50

$[\![\mathrm{op}]\!]_{\mathrm{eff}}(\sigma)(\sigma')$

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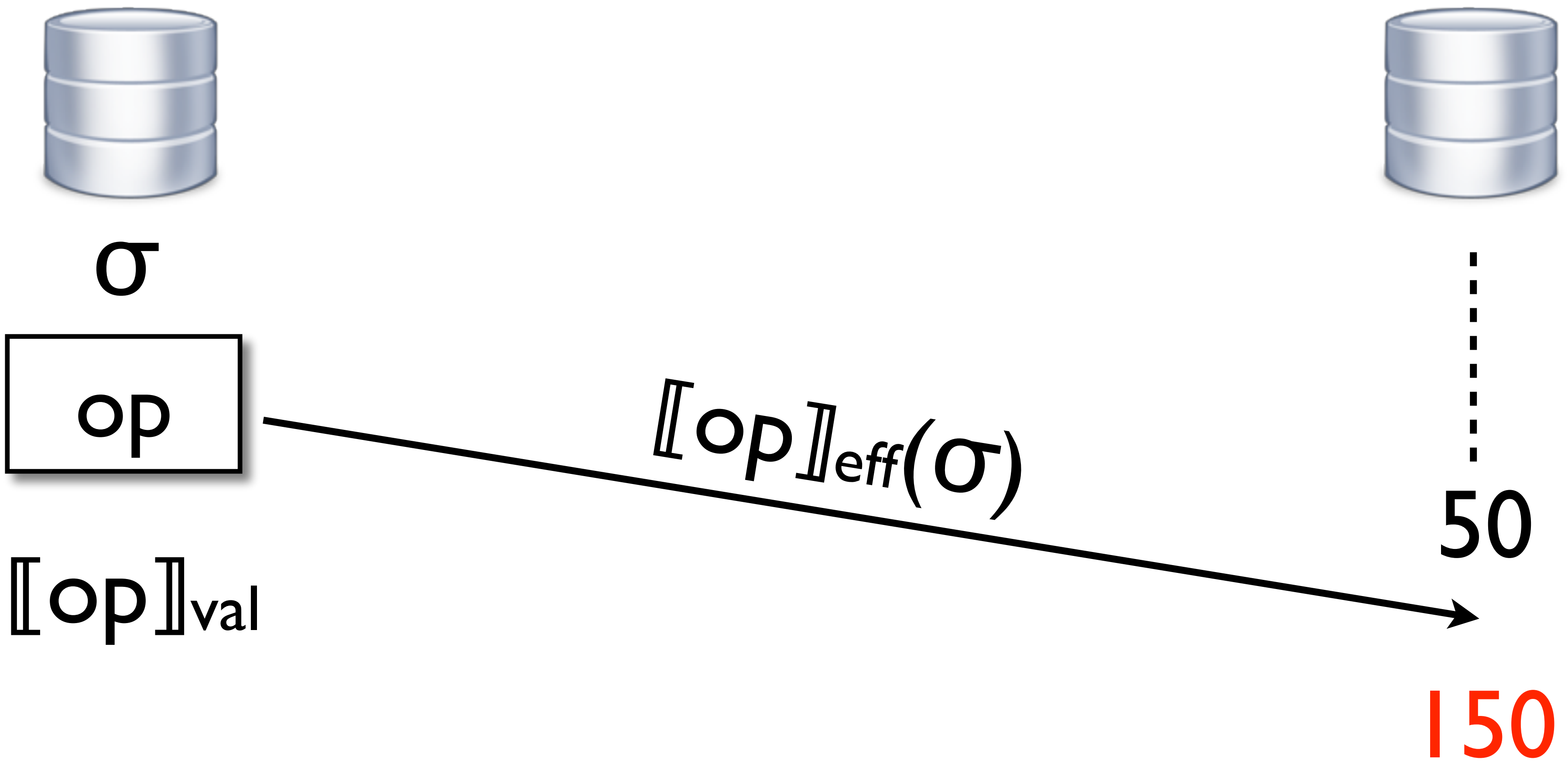
Operation semantics



σ

op

$[\![\text{op}]\!]_{\text{eff}}(\sigma)$

50

$[\![\text{op}]\!]_{\text{val}}$

150

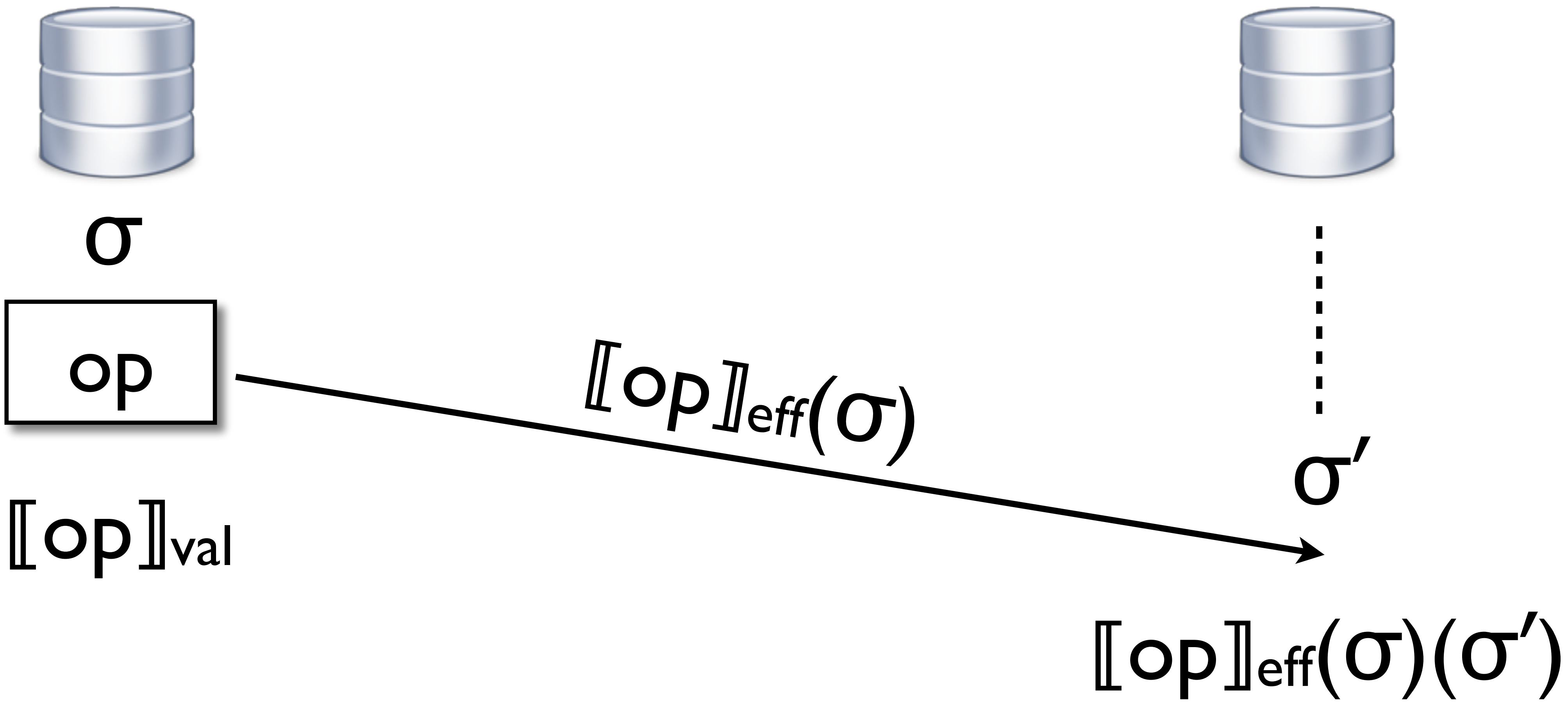
$$[\![\text{deposit}(100)]\!]_{\text{eff}}(\sigma) \;=\; \lambda\sigma'.\,(\sigma' + 100)$$

Ensuring eventual consistency

- Effectors have to commute
- Eventual consistency: replicas receiving the same messages in different orders end up in the same state
- Replicated data types [Shapiro+ 2011]: ready-made commutative implementations

$$[\![\mathrm{deposit}(100)]\!]_{\mathrm{eff}}(\sigma) \;=\; \lambda\sigma'.\,(\sigma' + 100)$$

Operation semantics



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op

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$\llbracket \mathrm{op} \rrbracket_{\mathrm{eff}}(\sigma)$

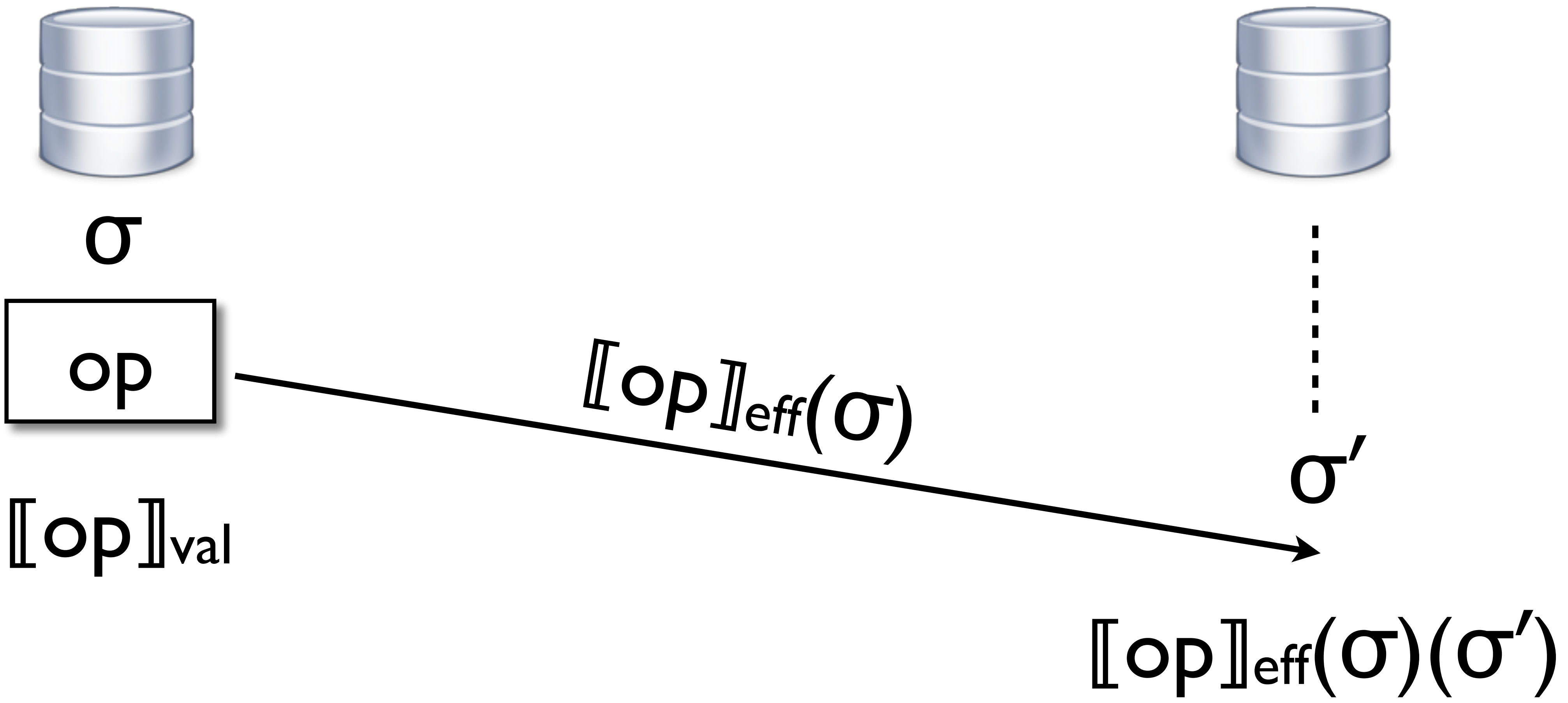
σ'

$\llbracket \mathrm{op} \rrbracket_{\mathrm{eff}}(\sigma)(\sigma')$

$\llbracket \mathrm{withdraw}(100) \rrbracket_{\mathrm{eff}}(\sigma) =$

$\quad \text{if } \sigma \geq 100 \text{ then } (\lambda\sigma'.\, \sigma' - 100) \text{ else } (\lambda\sigma'.\, \sigma')$

Operation semantics



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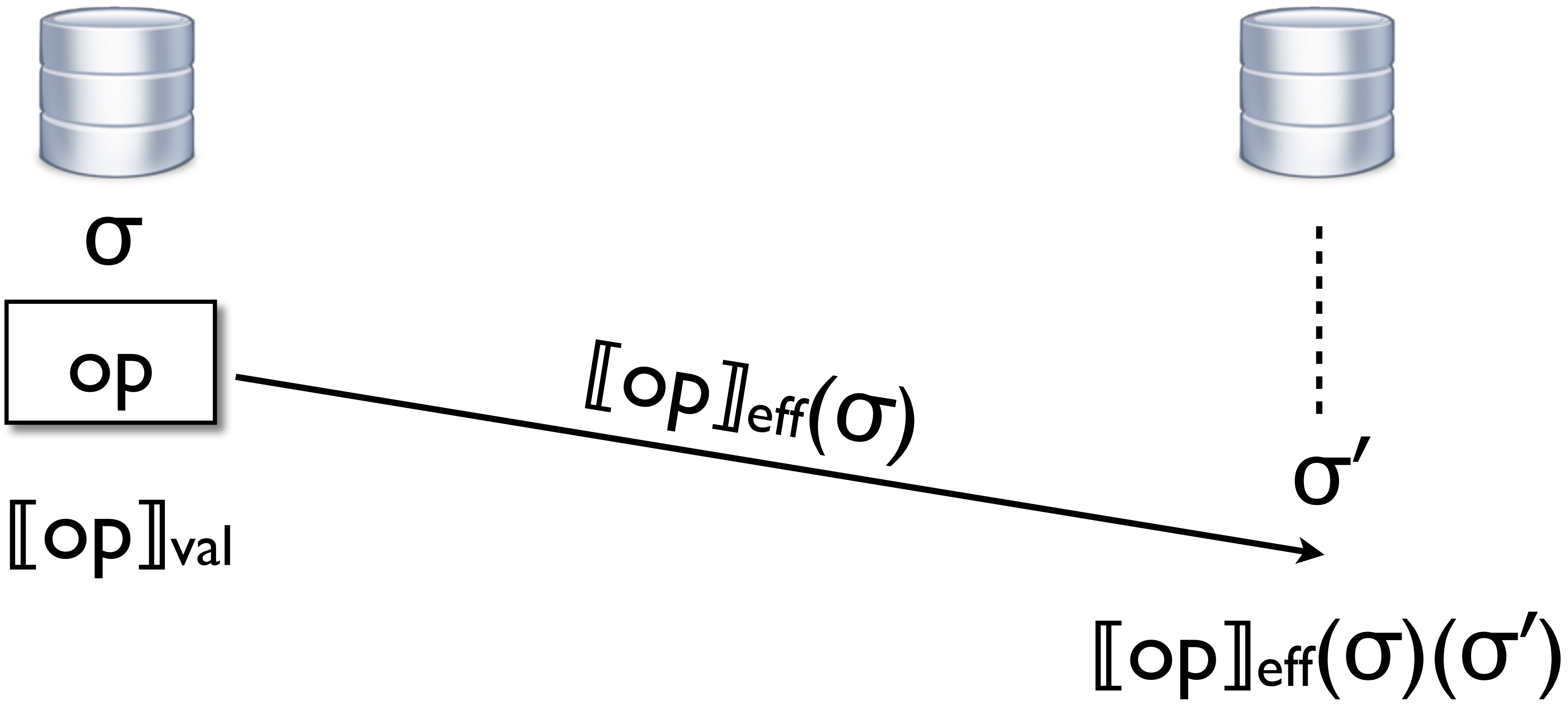

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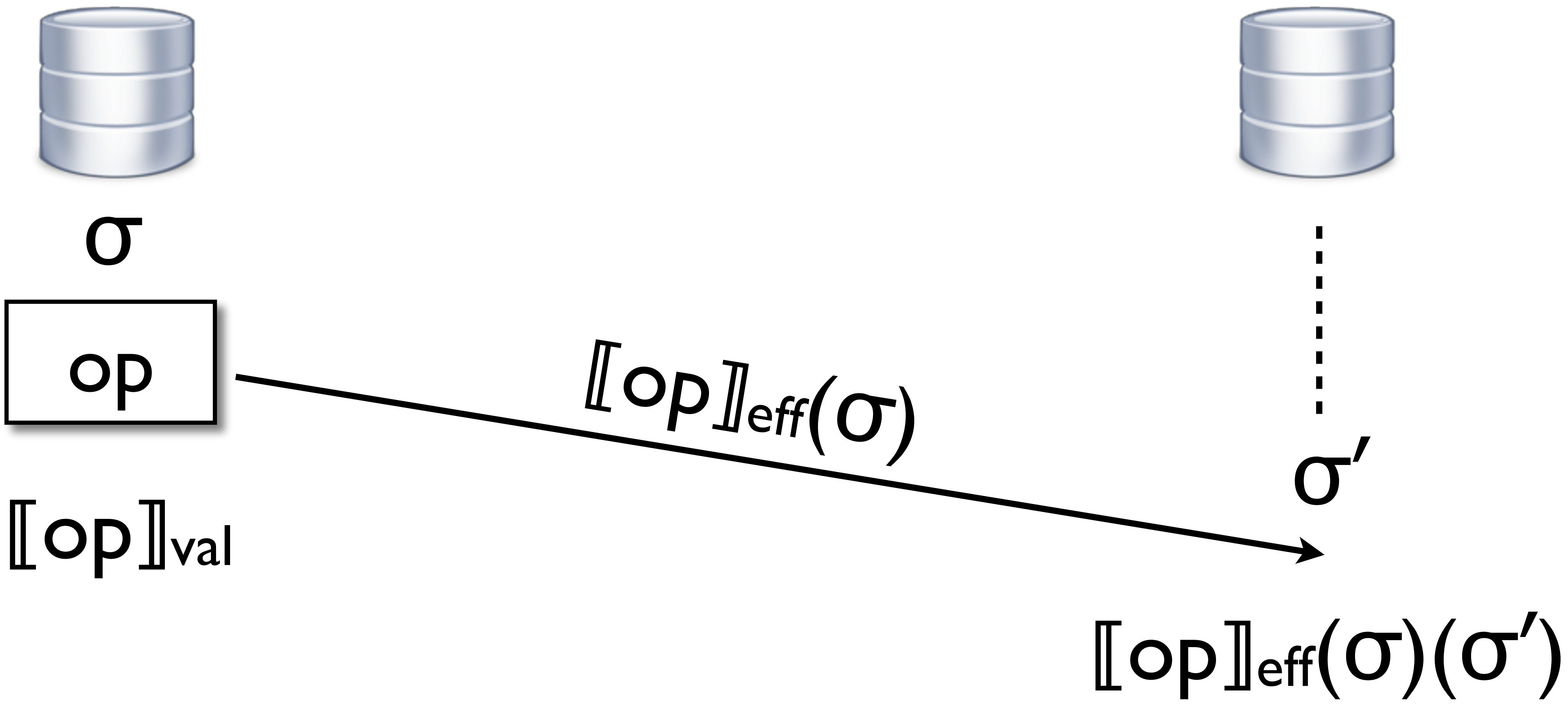

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$\llbracket op \rrbracket_{eff}(\sigma)$

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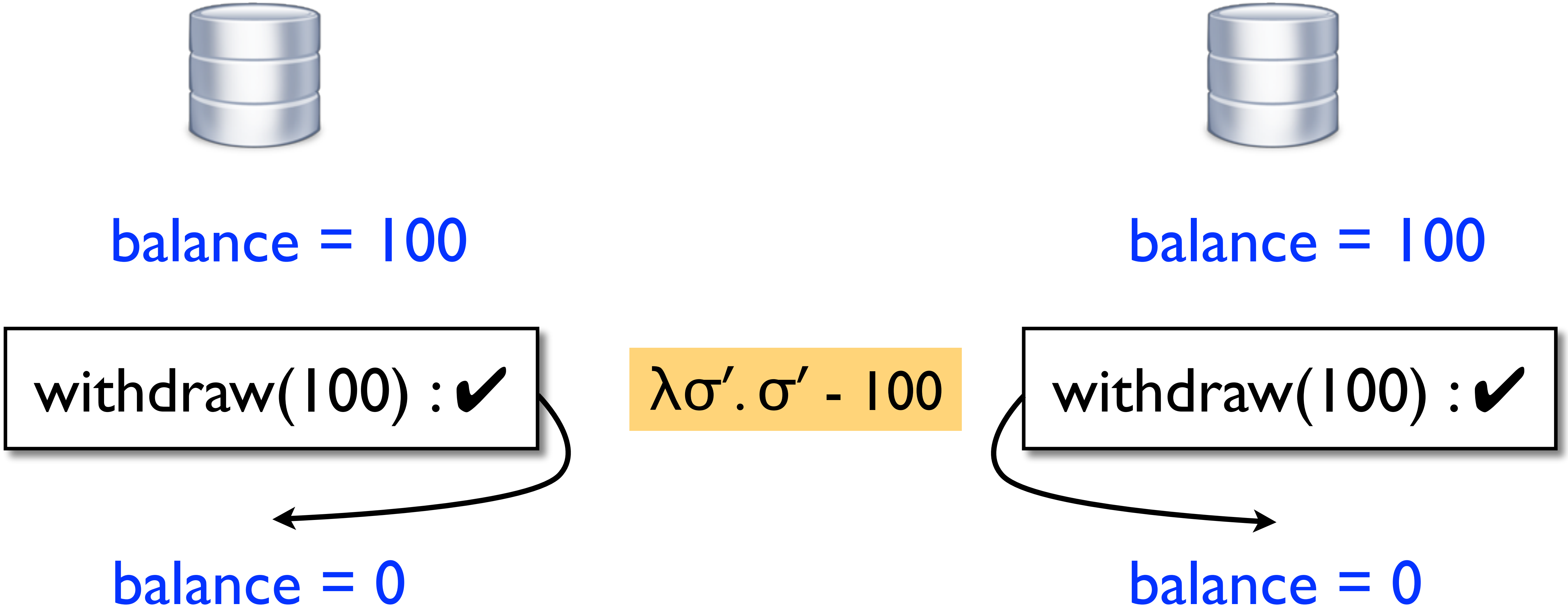

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withdraw(100) : ✔

balance = 0

$\lambda\sigma'.\ \sigma' - 100$

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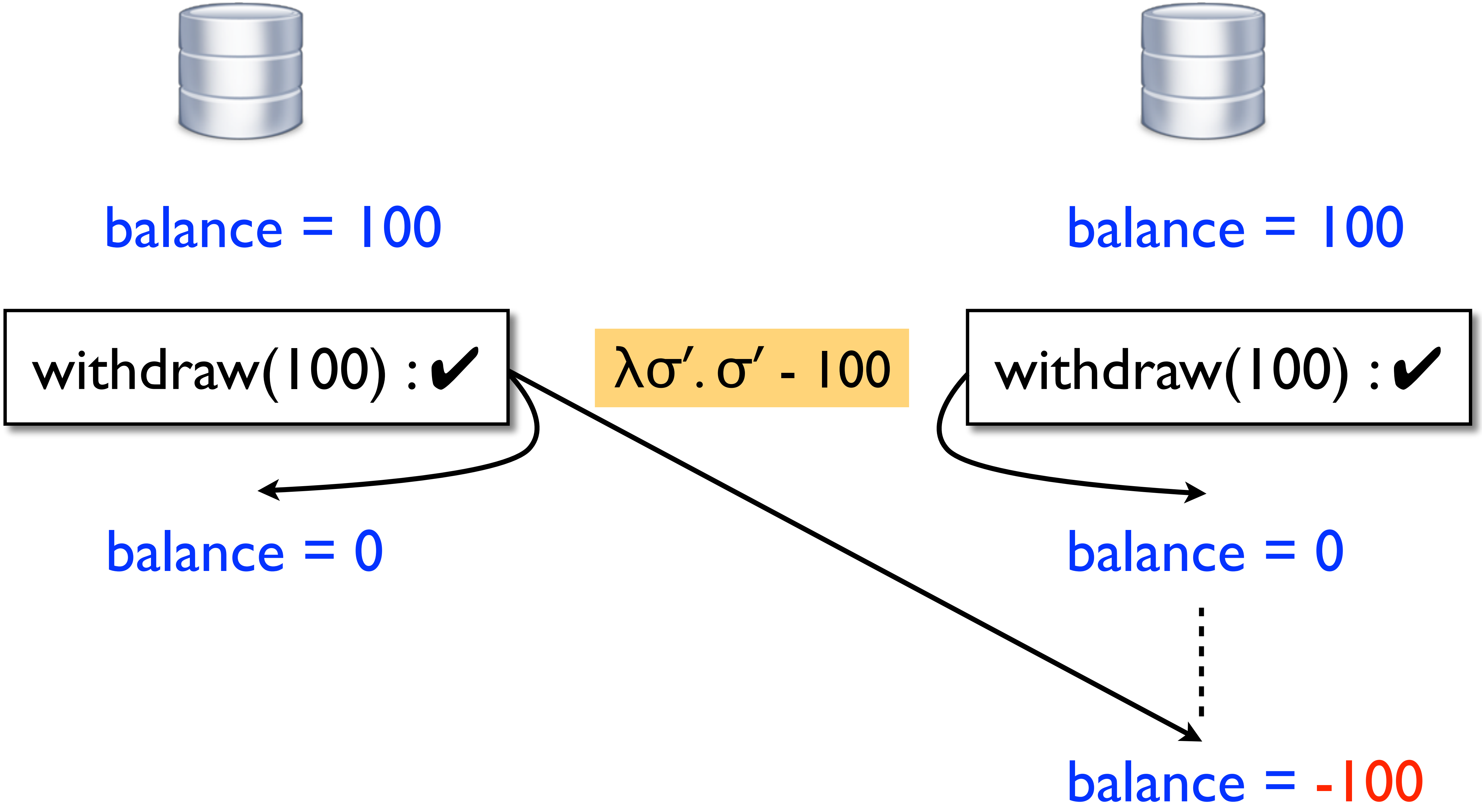

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Strengthening consistency

Token system ≈ locks on steroids:

- Token = $\{\tau_1, \tau_2, \ldots\}$
- Symmetric conflict relation $\bowtie \subseteq \text{Token} \times \text{Token}$

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Example - mutual exclusion lock:
Token = $\{\tau\}$; $\tau \bowtie \tau$

Strengthening consistency

Token system ≈ locks on steroids:

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- Symmetric conflict relation $\bowtie \subseteq \text{Token} \times \text{Token}$

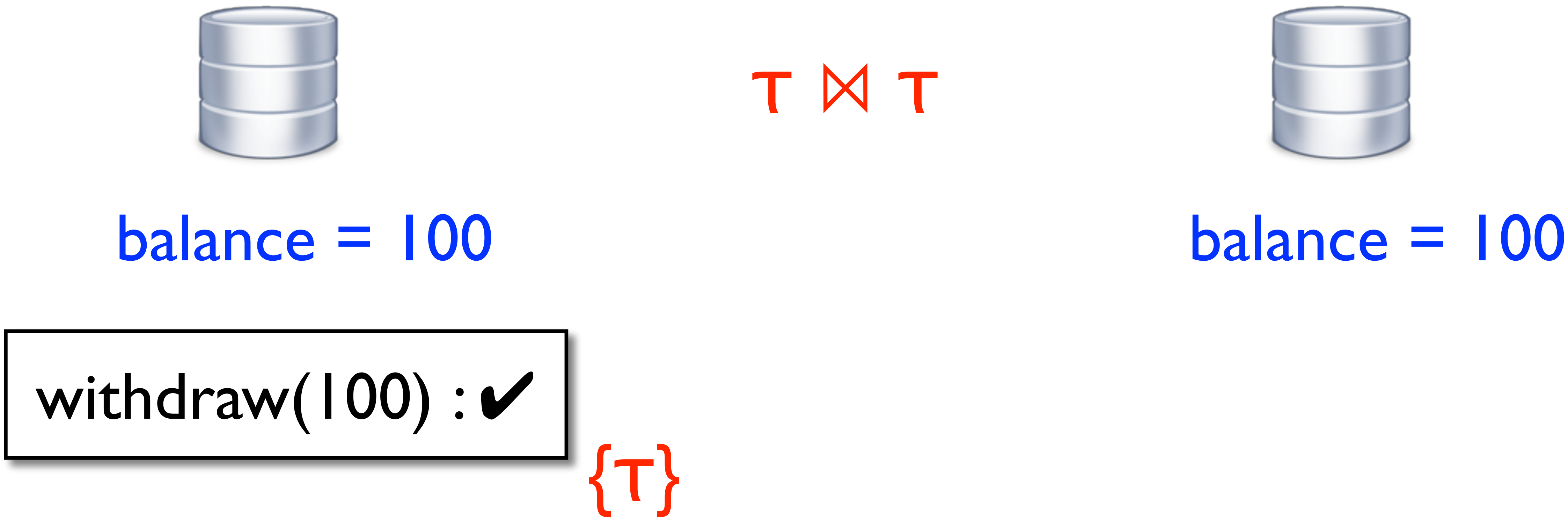
Example - mutual exclusion lock:

$\text{Token} = \{\tau\};\ \tau \bowtie \tau$

Each operation associated with a set of tokens:

$[\![\text{op}]\!]_{\text{tok}} \in \text{State} \rightarrow \mathcal{P}(\text{Token})$

Operations associated with conflicting tokens cannot be unaware of each other



$\tau \bowtie \tau$

balance = 100

balance = 100

withdraw(100) : ✔

$\{\tau\}$

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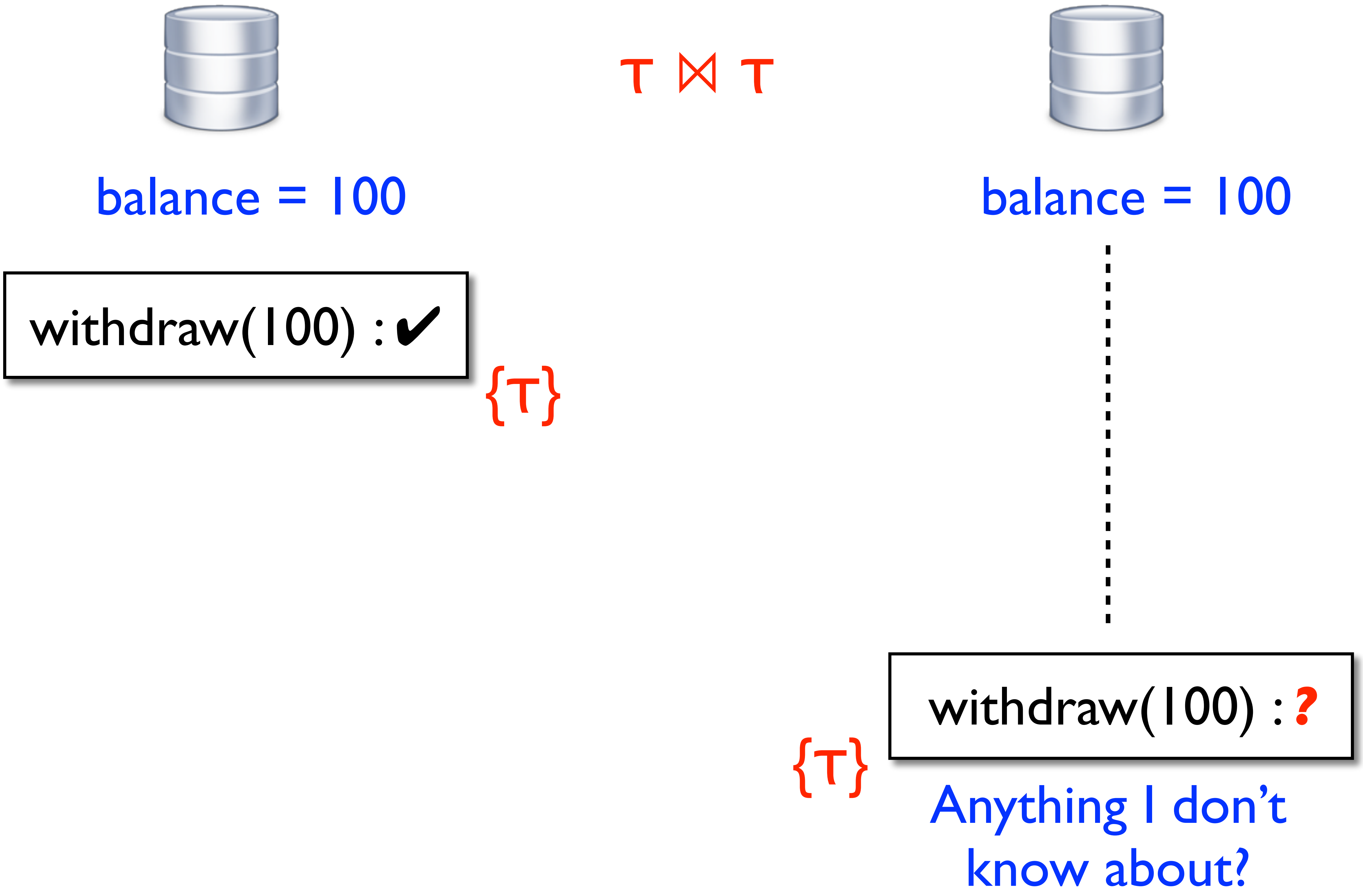


$\tau \bowtie \tau$

balance = 100

balance = 100

withdraw(100) : ✔

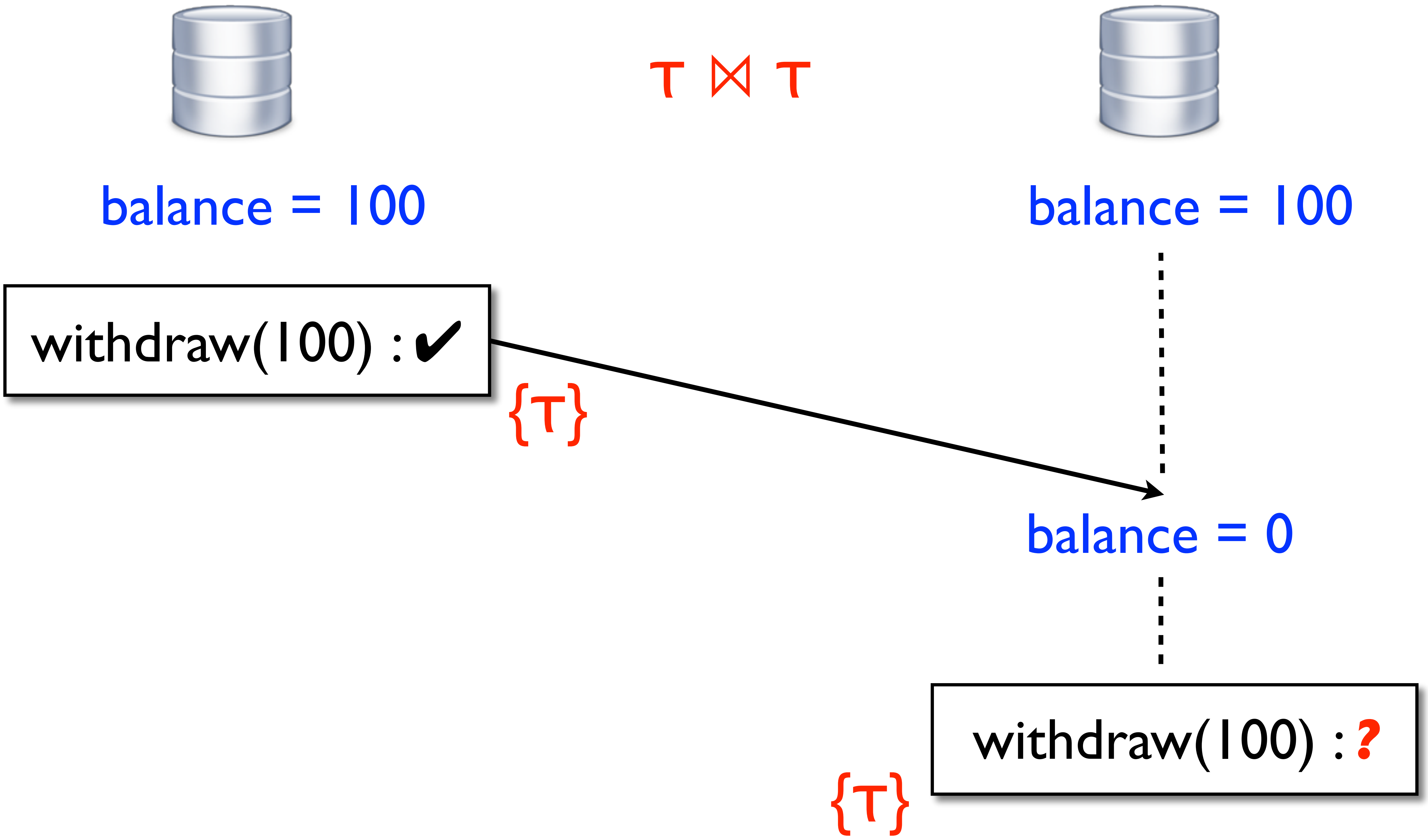
{τ}

{τ}

withdraw(100) : ?

Anything I don’t know about?

Operations associated with conflicting tokens cannot be unaware of each other



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{τ}

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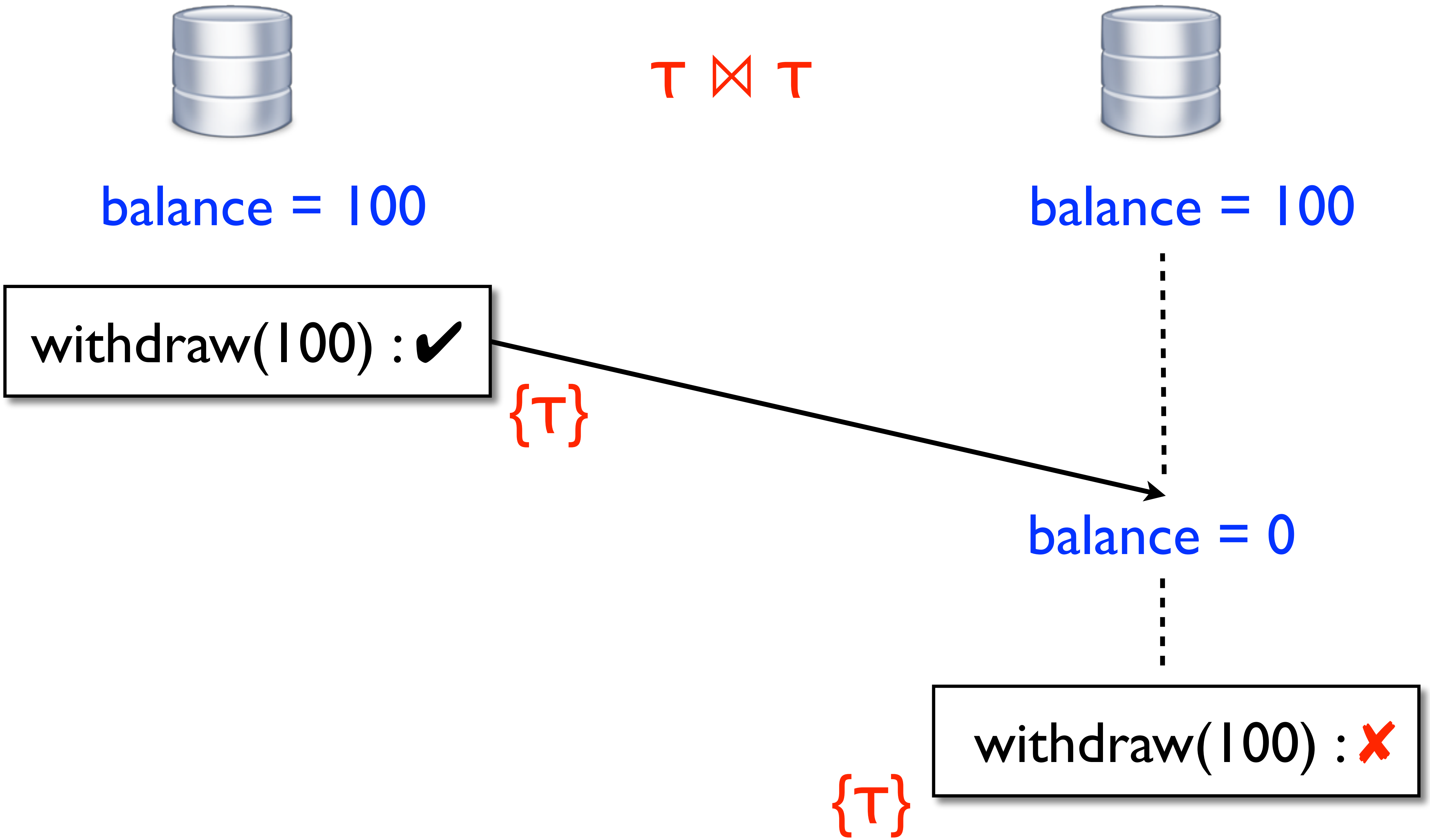

$\tau \bowtie \tau$

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{τ}

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withdraw(100) : ✘

{τ}

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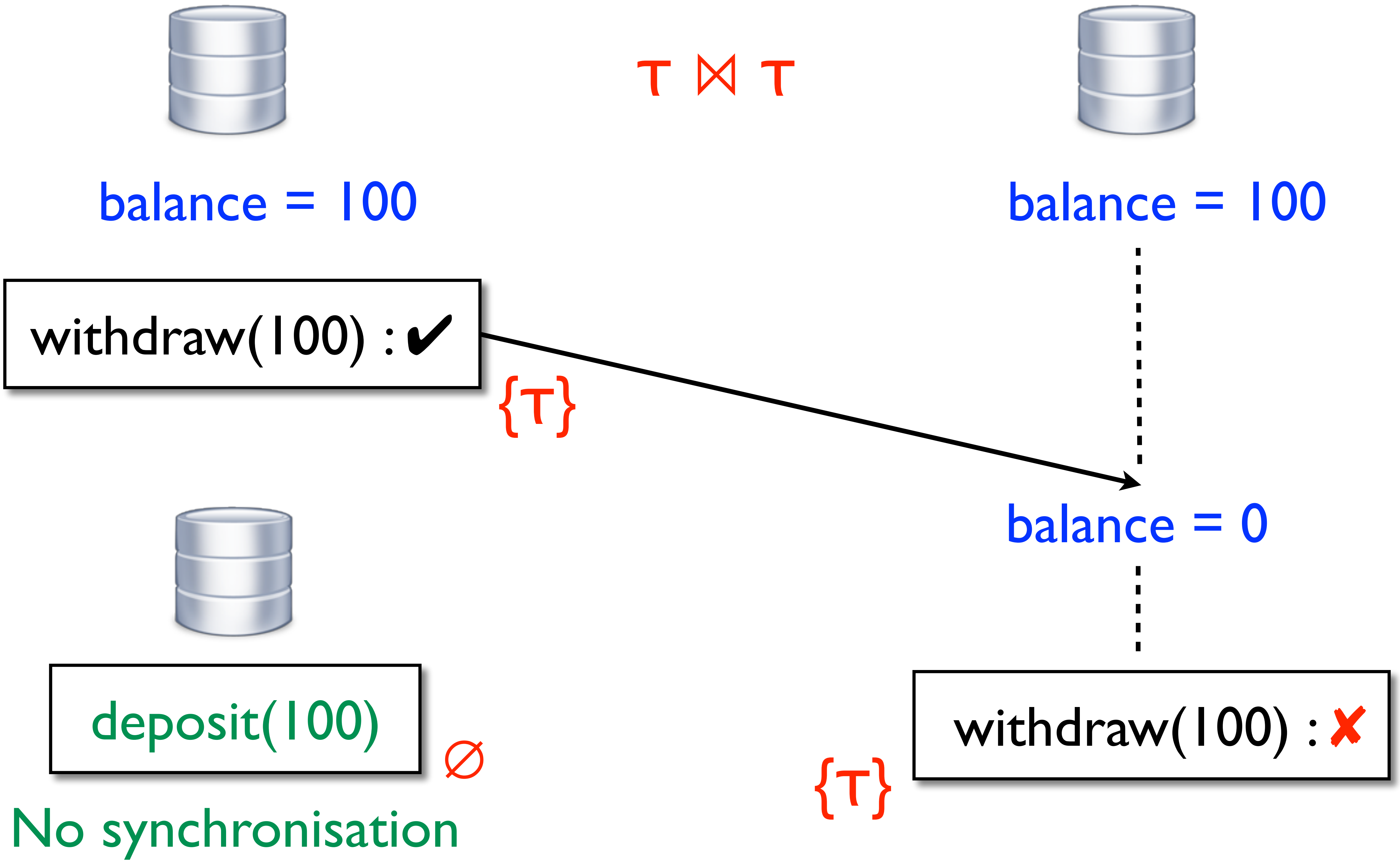


$\tau \bowtie \tau$

balance = 100

balance = 100

withdraw(100) : ✔

$\{\tau\}$

balance = 0

deposit(100)

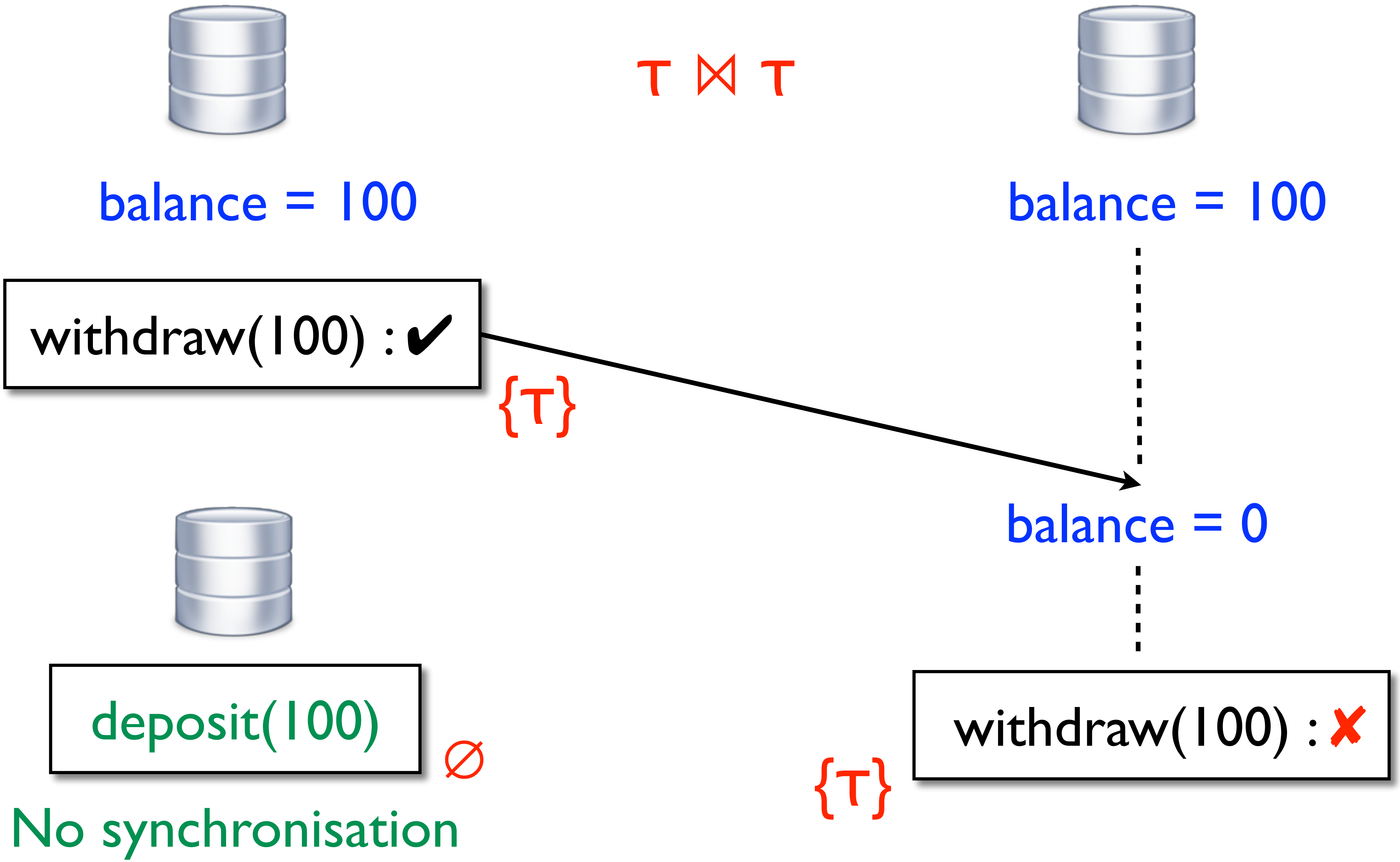
$\varnothing$

No synchronisation

$\{\tau\}$

withdraw(100) : ✘

Do we always have I = (balance ≥ 0)?



$\top \bowtie \top$

balance = 100

balance = 100

withdraw(100) : ✔

$\{\top\}$

balance = 0

deposit(100)

$\varnothing$

No synchronisation

$\{\top\}$

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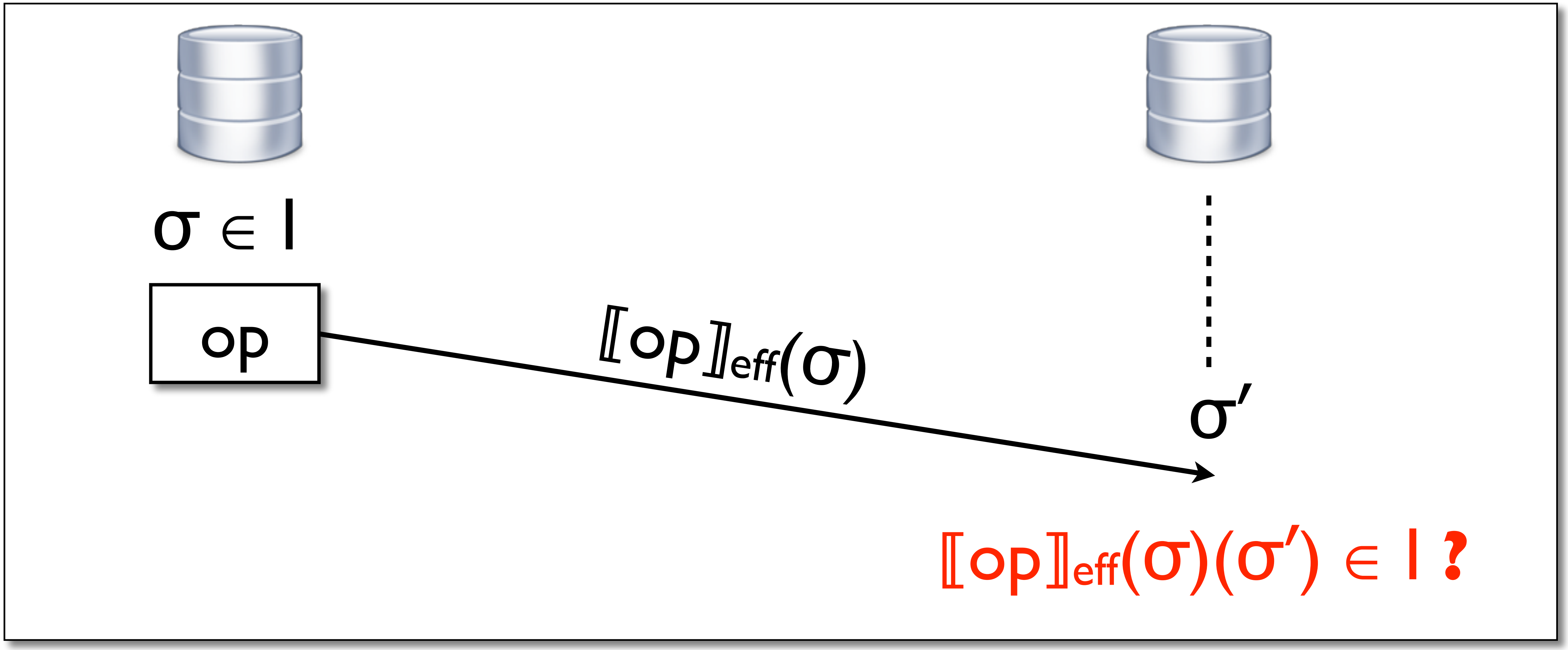


$\sigma \in I$

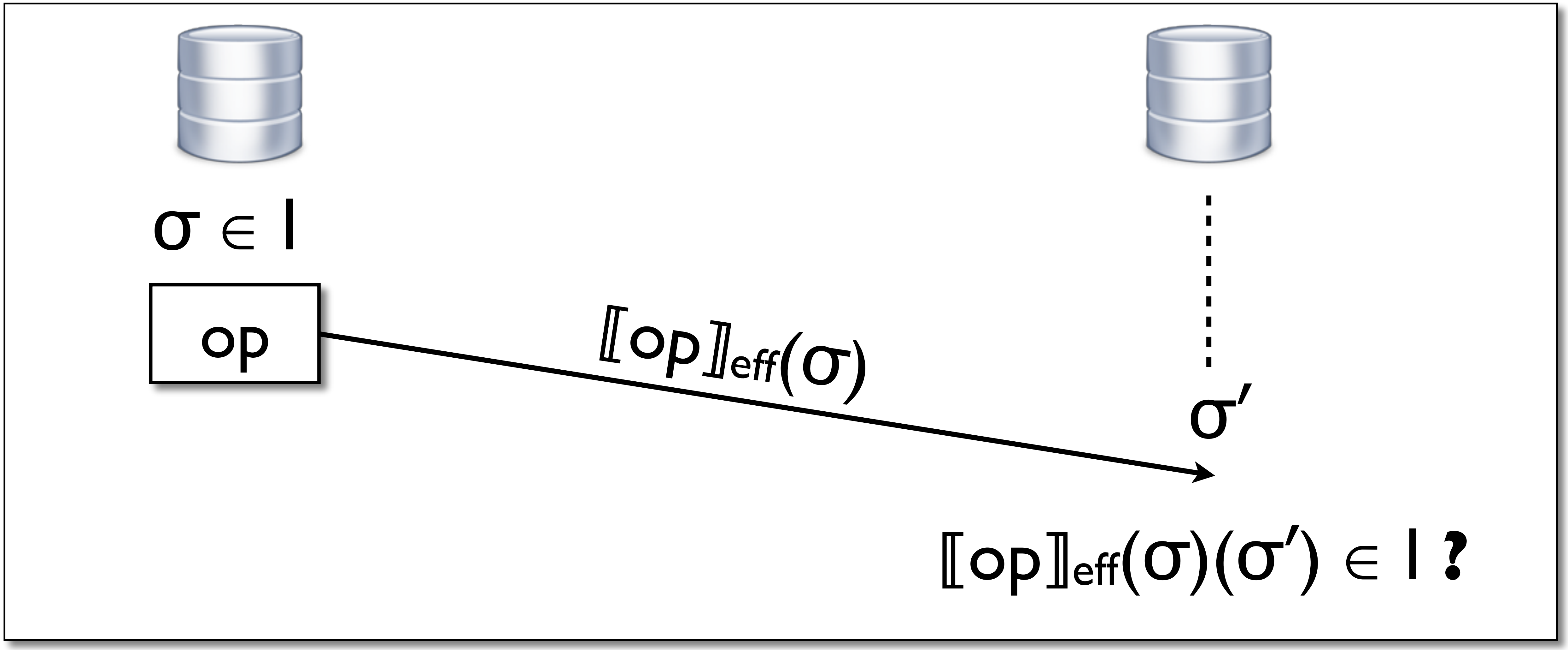
op

$[\![op]\!]_{eff}(\sigma)$

σ'

$[\![op]\!]_{eff}(\sigma)(\sigma') \in I$?

Effect applied in a different state!



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$\llbracket \mathrm{op} \rrbracket_{\mathrm{eff}}(\sigma)$

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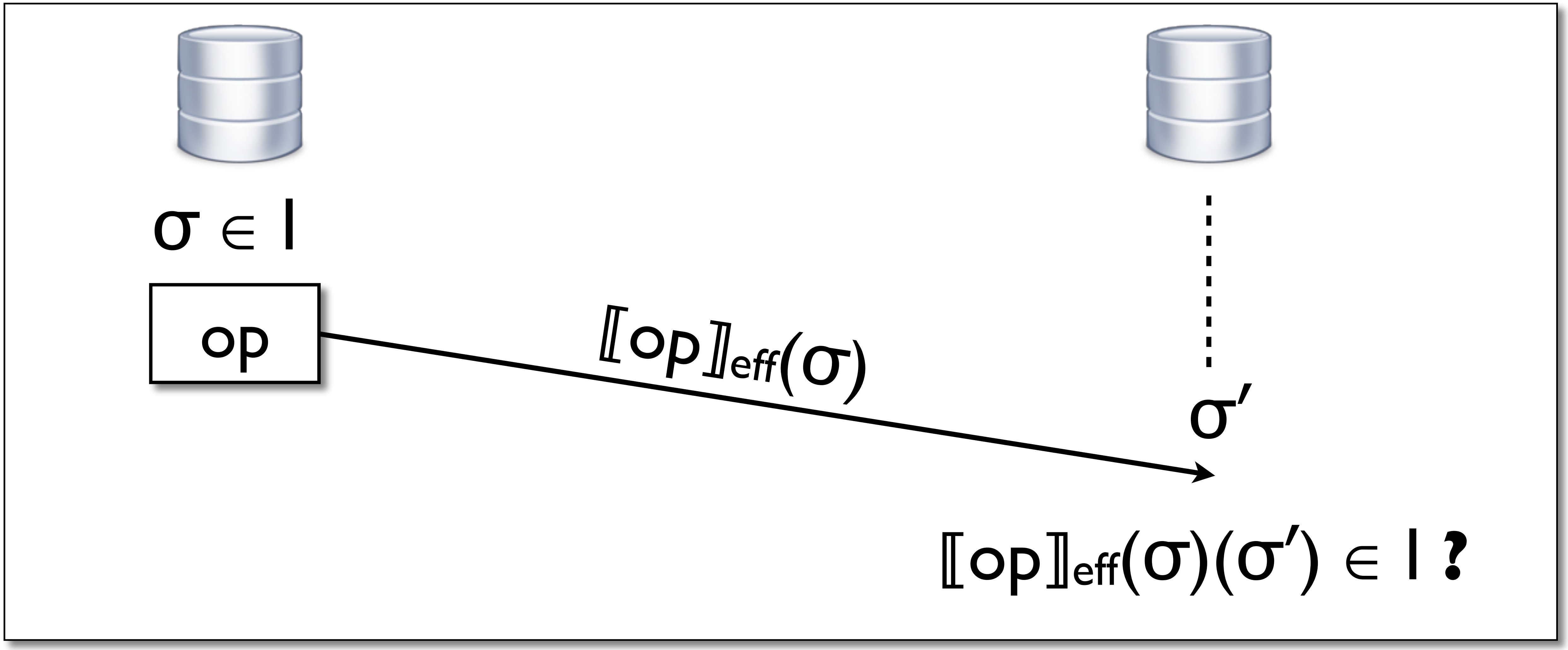
$\llbracket \mathrm{op} \rrbracket_{\mathrm{eff}}(\sigma)(\sigma') \in I$?

$\llbracket \mathrm{op} \rrbracket_{\mathrm{eff}}(\sigma) =$ if $P(\sigma)$ then $f(\sigma)$ else if...

$\llbracket \mathrm{withdraw}(100) \rrbracket_{\mathrm{eff}}(\sigma) =$

if $\sigma \geq 100$ then $(\lambda\sigma'.\,\sigma' - 100)$ else $(\lambda\sigma'.\,\sigma')$



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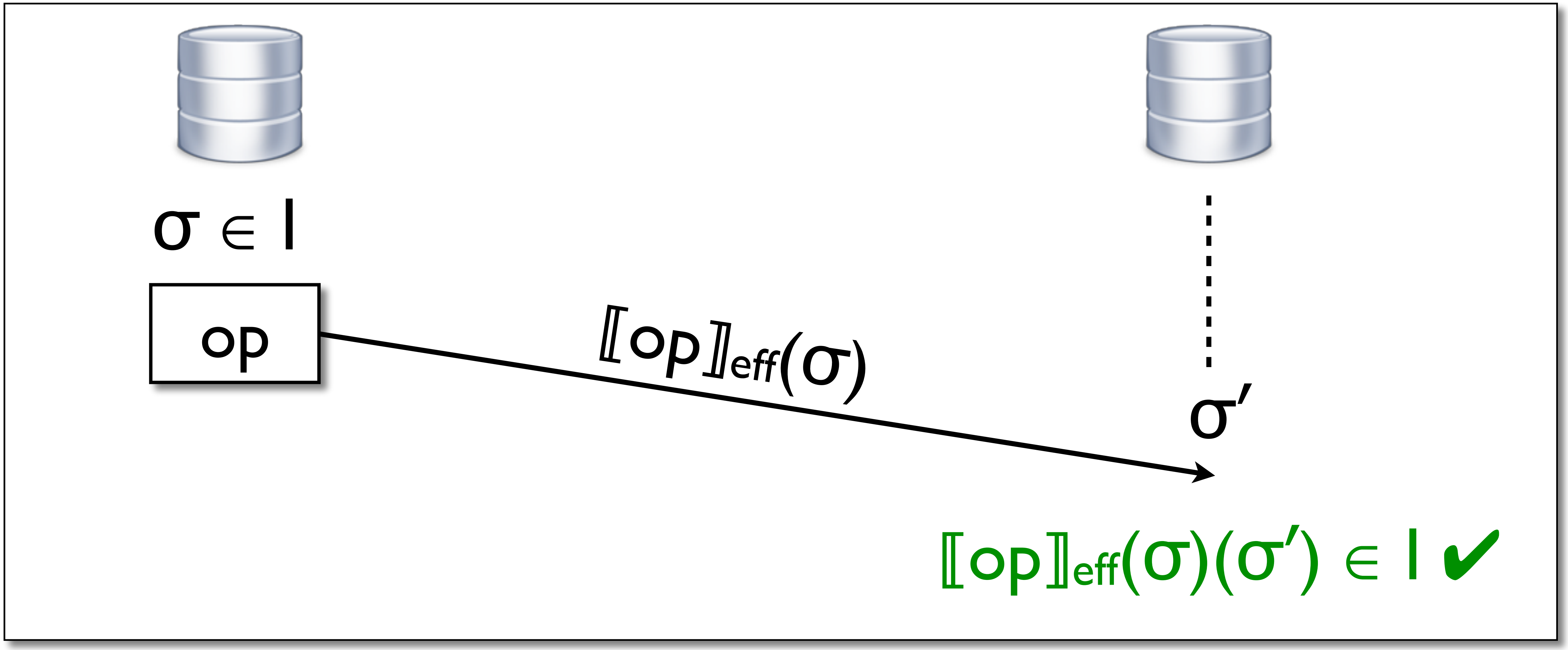
$[\![op]\!]_{eff}(\sigma)$

σ'

$[\![op]\!]_{eff}(\sigma)(\sigma') \in I$ **?**

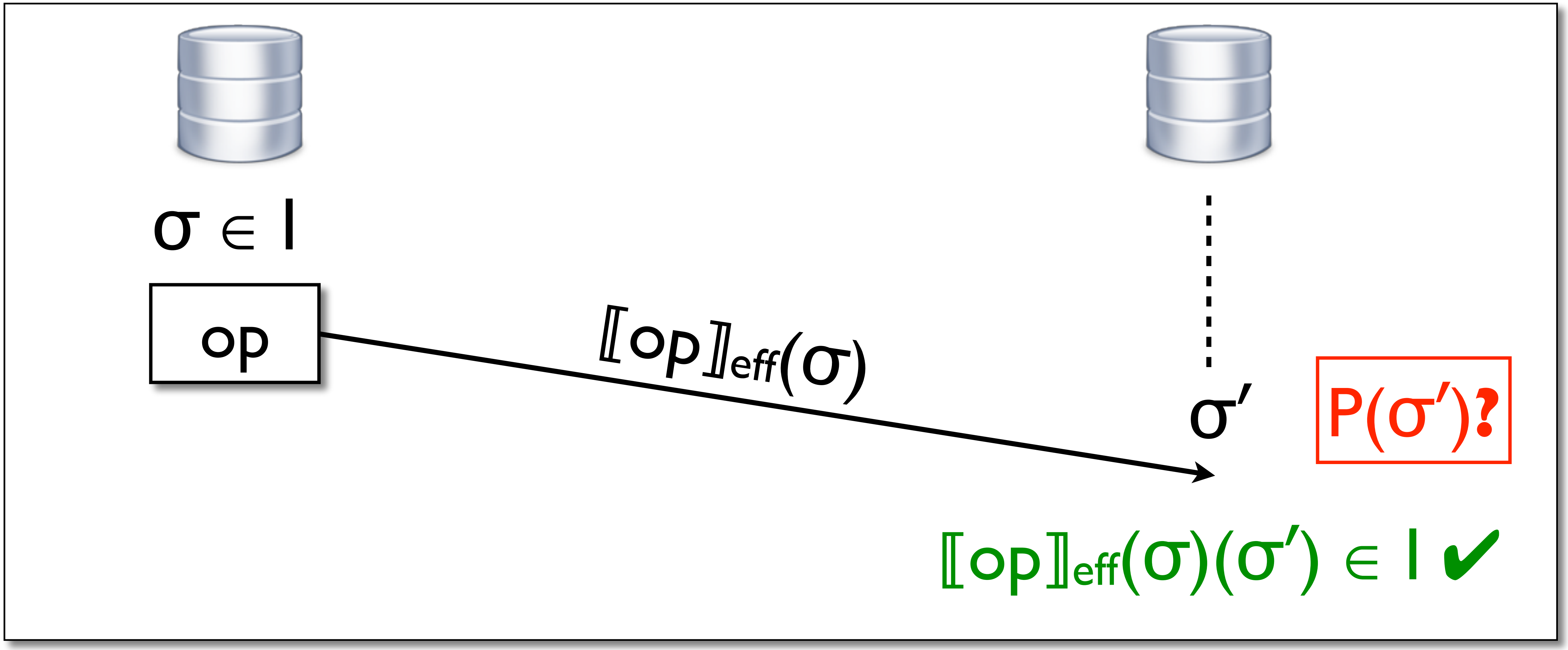
$[\![op]\!]_{eff}(\sigma)$ = if $P(\sigma)$ then $f(\sigma)$ else if...

1. Effector safety: f preserves I when executed in any state satisfying P



$[\![op]\!]_{eff}(\sigma) = \text{if } P(\sigma) \text{ then } f(\sigma) \text{ else if...}$

1. **Effector safety:** f preserves I when executed in any state satisfying P



$\sigma \in I$

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$[\![op]\!]_{eff}(\sigma)$

σ'

$P(\sigma')$?

$[\![op]\!]_{eff}(\sigma)(\sigma') \in I$ ✔

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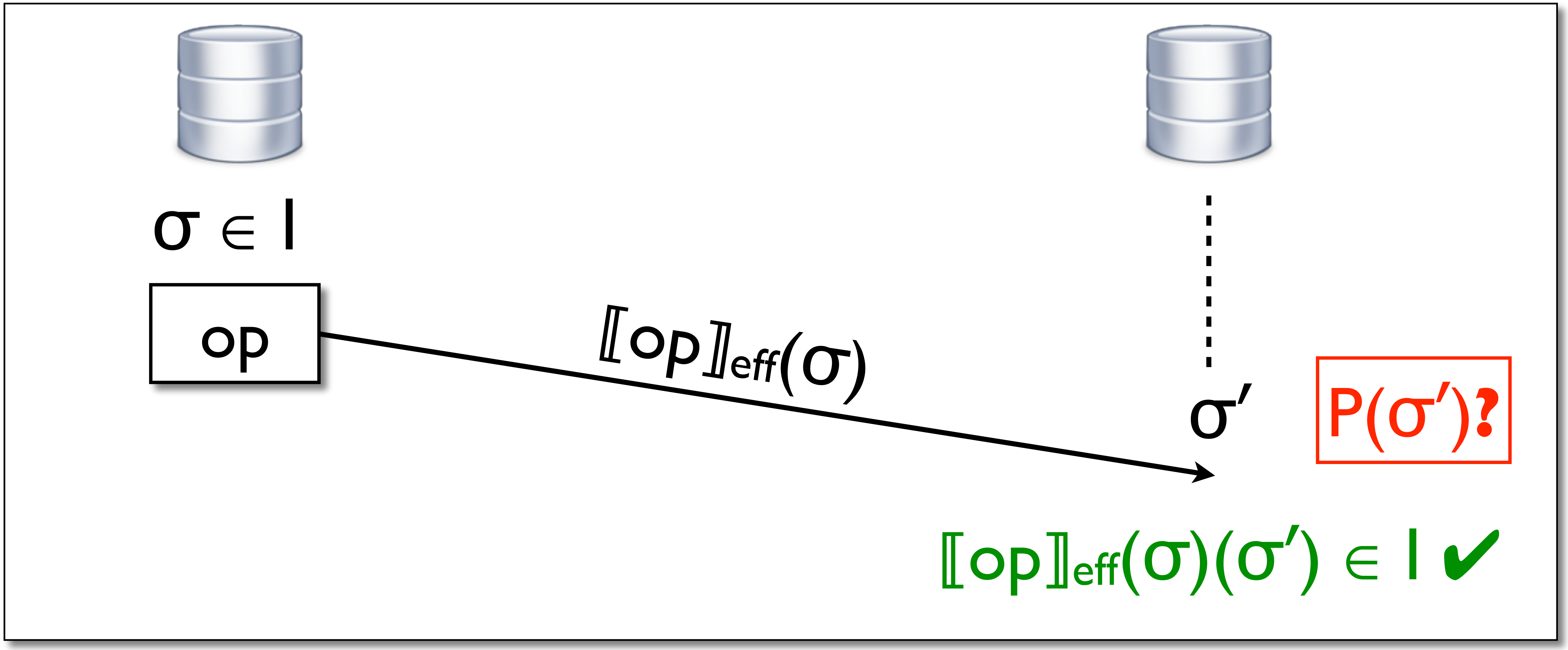

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1. Effector safety: f preserves I when executed in any state satisfying P

2. Precondition stability: P will hold when f is applied at any replica

CISE tool: ‘Cause I’m Strong Enough

Discharges proof obligations using Z3 SMT solver

By Mahsa Najafzadeh (UPMC & INRIA)

$[\![op]\!]_{eff}(\sigma)$ = if $P(\sigma)$ then $f(\sigma)$ else if...

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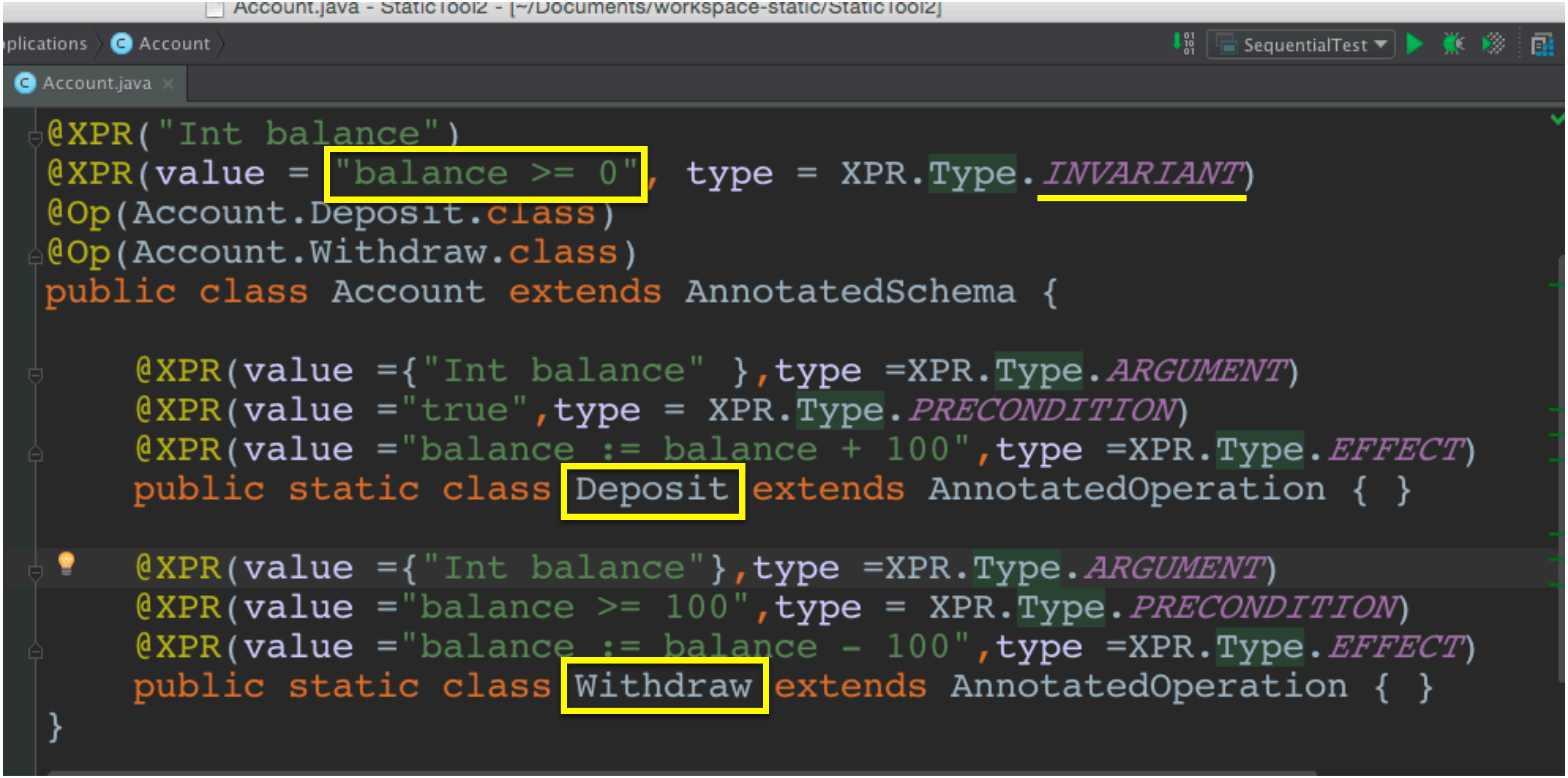
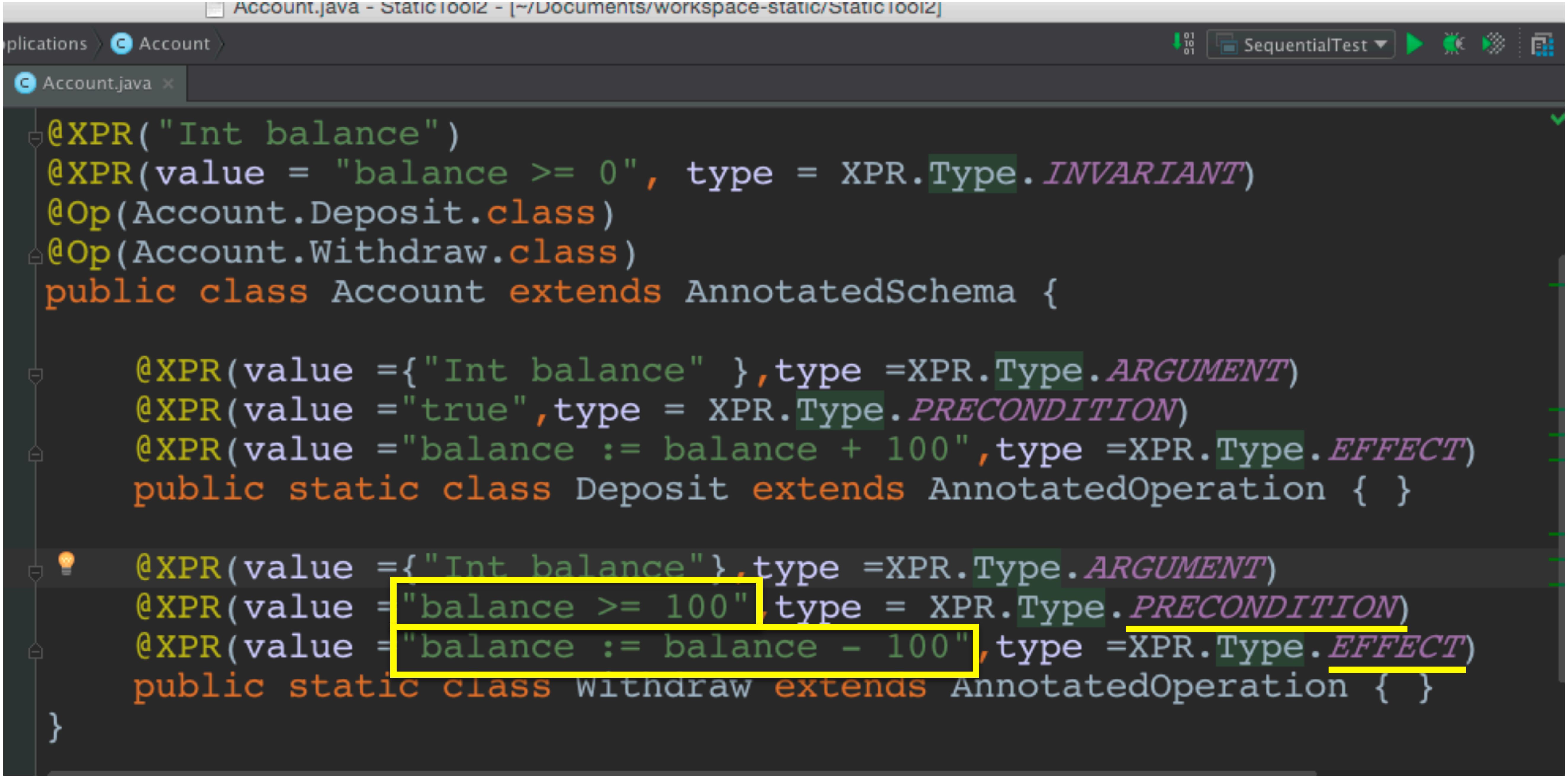
2. Precondition stability: P will hold when f is applied at any replica



```
@XPR("Int balance")
@XPR(value = "balance >= 0", type = XPR.Type.INVARIANT)
@Op(Account.Deposit.class)
@Op(Account.Withdraw.class)
public class Account extends AnnotatedSchema {

    @XPR(value ={"Int balance" },type =XPR.Type.ARGUMENT)
    @XPR(value ="true",type = XPR.Type.PRECONDITION)
    @XPR(value ="balance := balance + 100",type =XPR.Type.EFFECT)
    public static class Deposit extends AnnotatedOperation { }

    @XPR(value ={"Int balance"},type =XPR.Type.ARGUMENT)
    @XPR(value ="balance >= 100",type = XPR.Type.PRECONDITION)
    @XPR(value ="balance := balance - 100",type =XPR.Type.EFFECT)
    public static class Withdraw extends AnnotatedOperation { }
}
```



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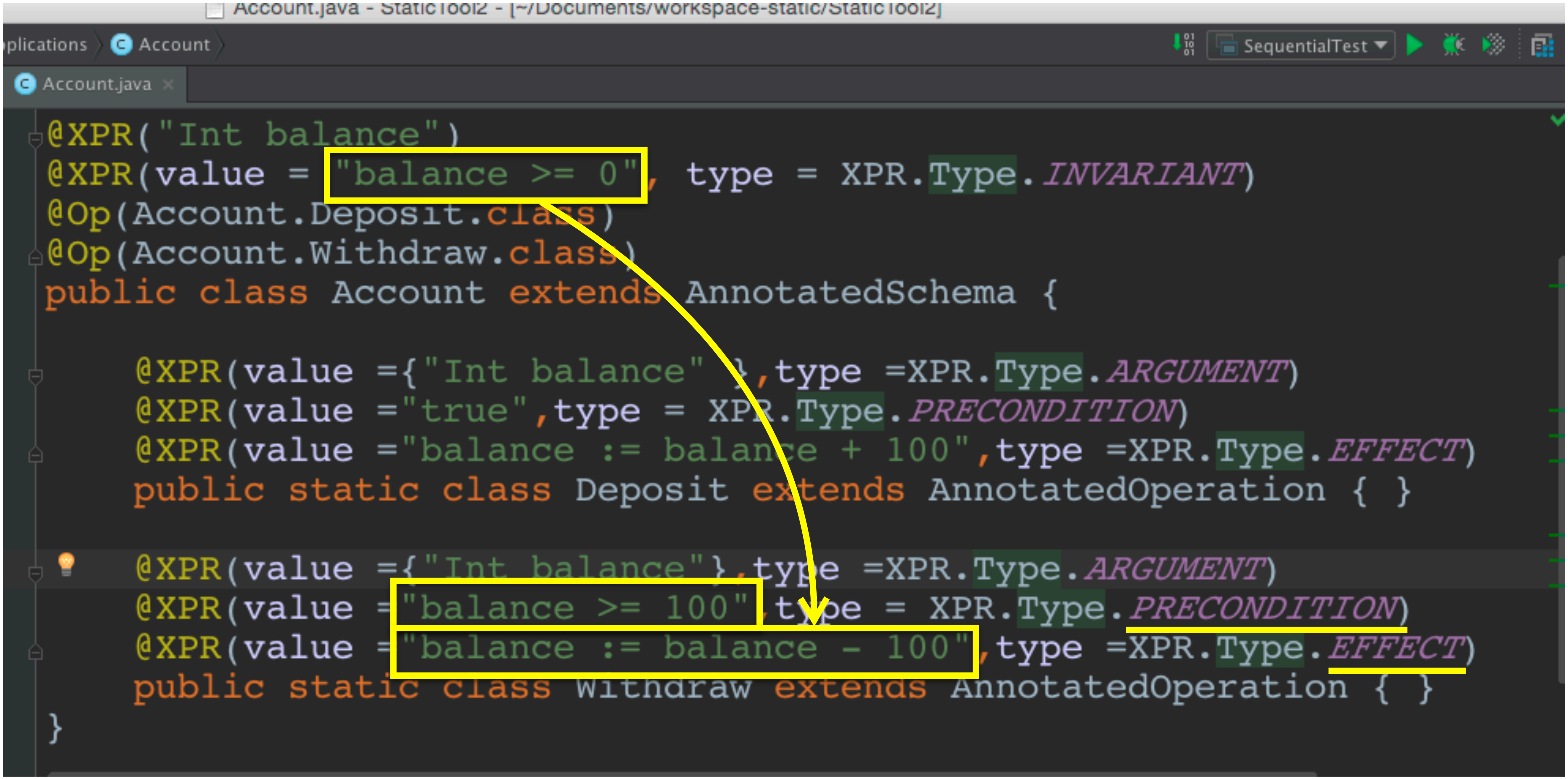
1. Effector safety: f preserves I when executed in any state satisfying P



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@XPR("Int balance")
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@Op(Account.Deposit.class)
@Op(Account.Withdraw.class)
public class Account extends AnnotatedSchema {

    @XPR(value ={"Int balance" },type =XPR.Type.ARGUMENT)
    @XPR(value ="true",type = XPR.Type.PRECONDITION)
    @XPR(value ="balance := balance + 100",type =XPR.Type.EFFECT)
    public static class Deposit extends AnnotatedOperation { }

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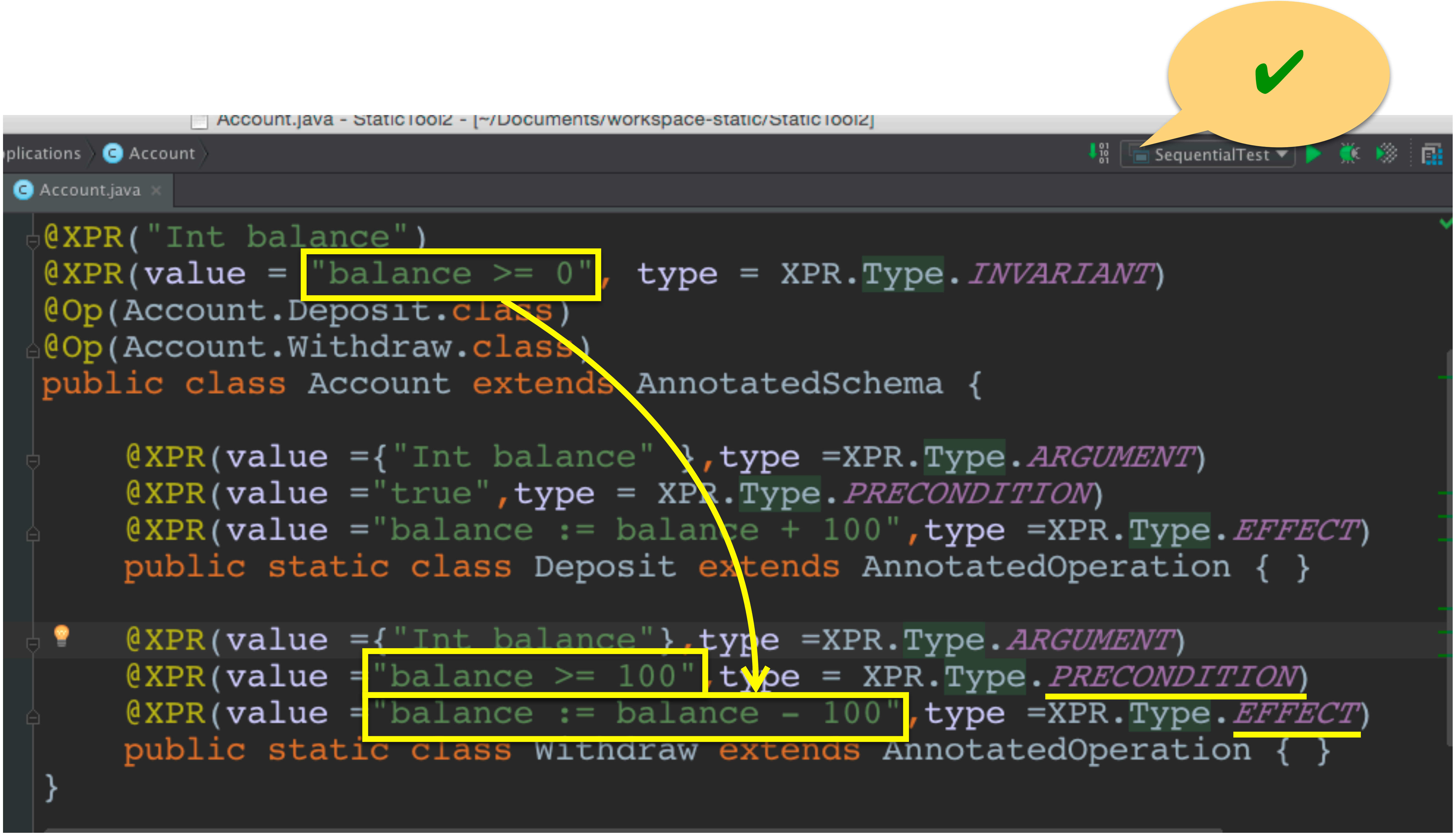

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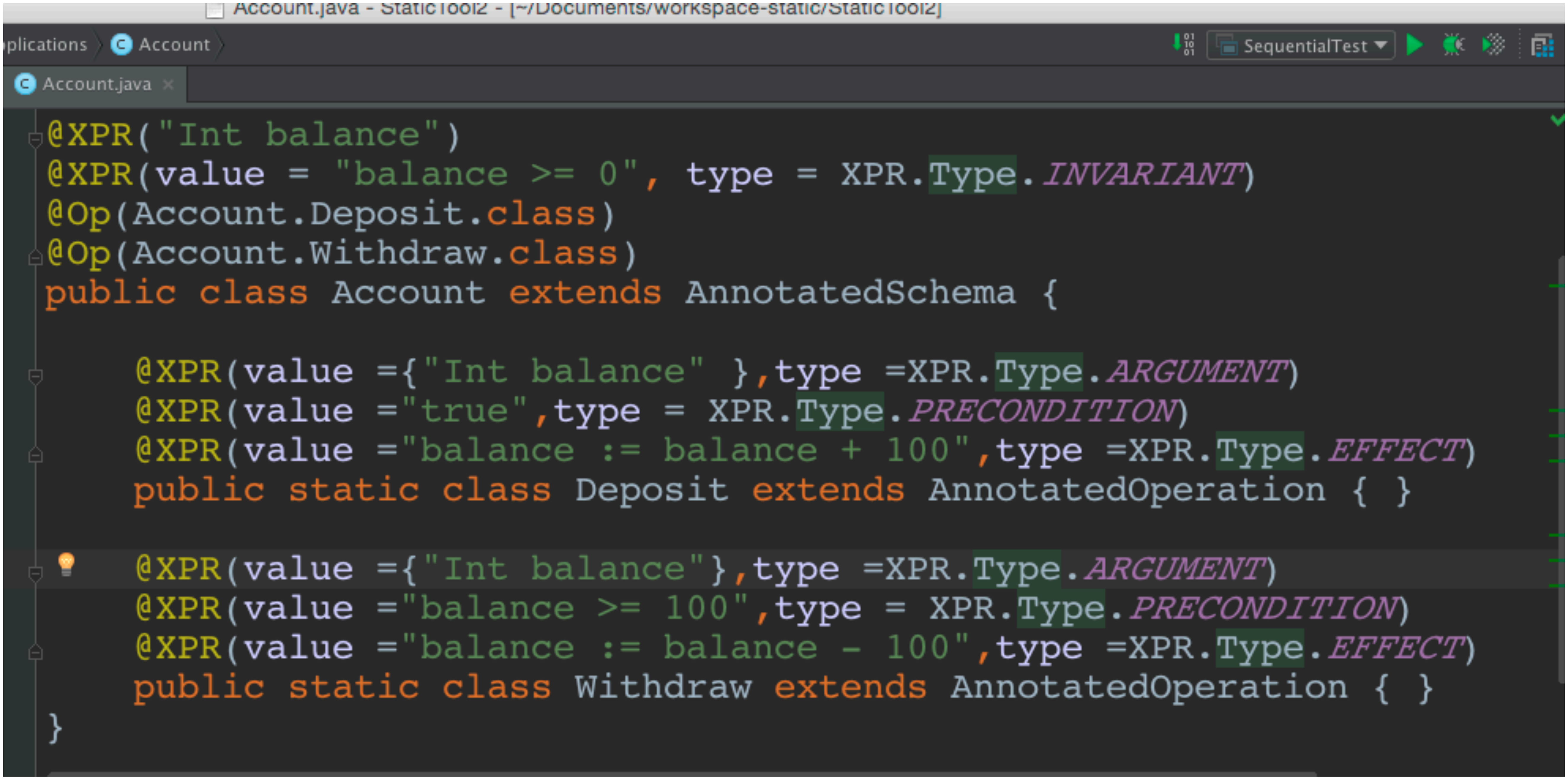
2. Precondition stability: P is preserved by concurrent operations



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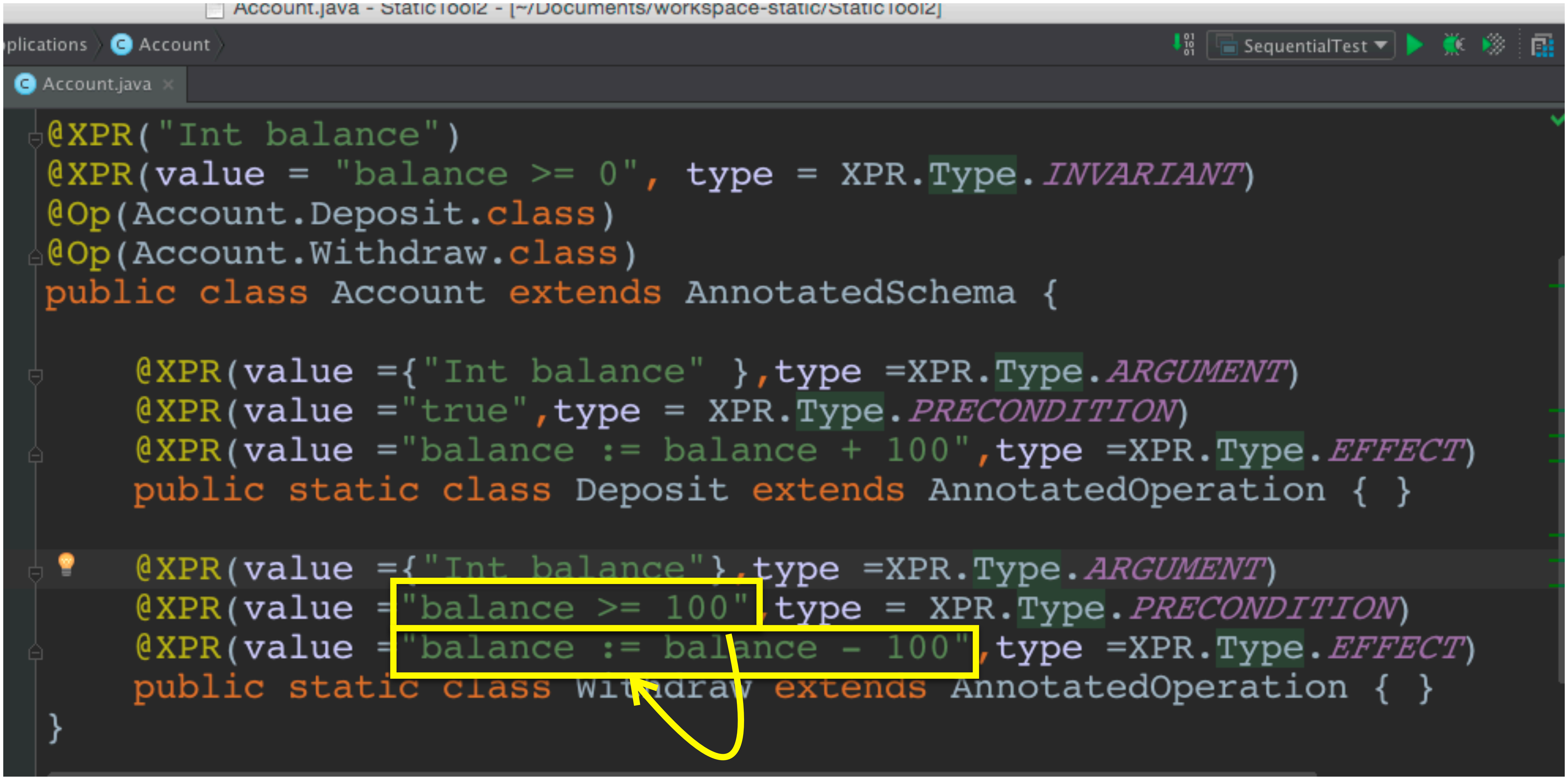

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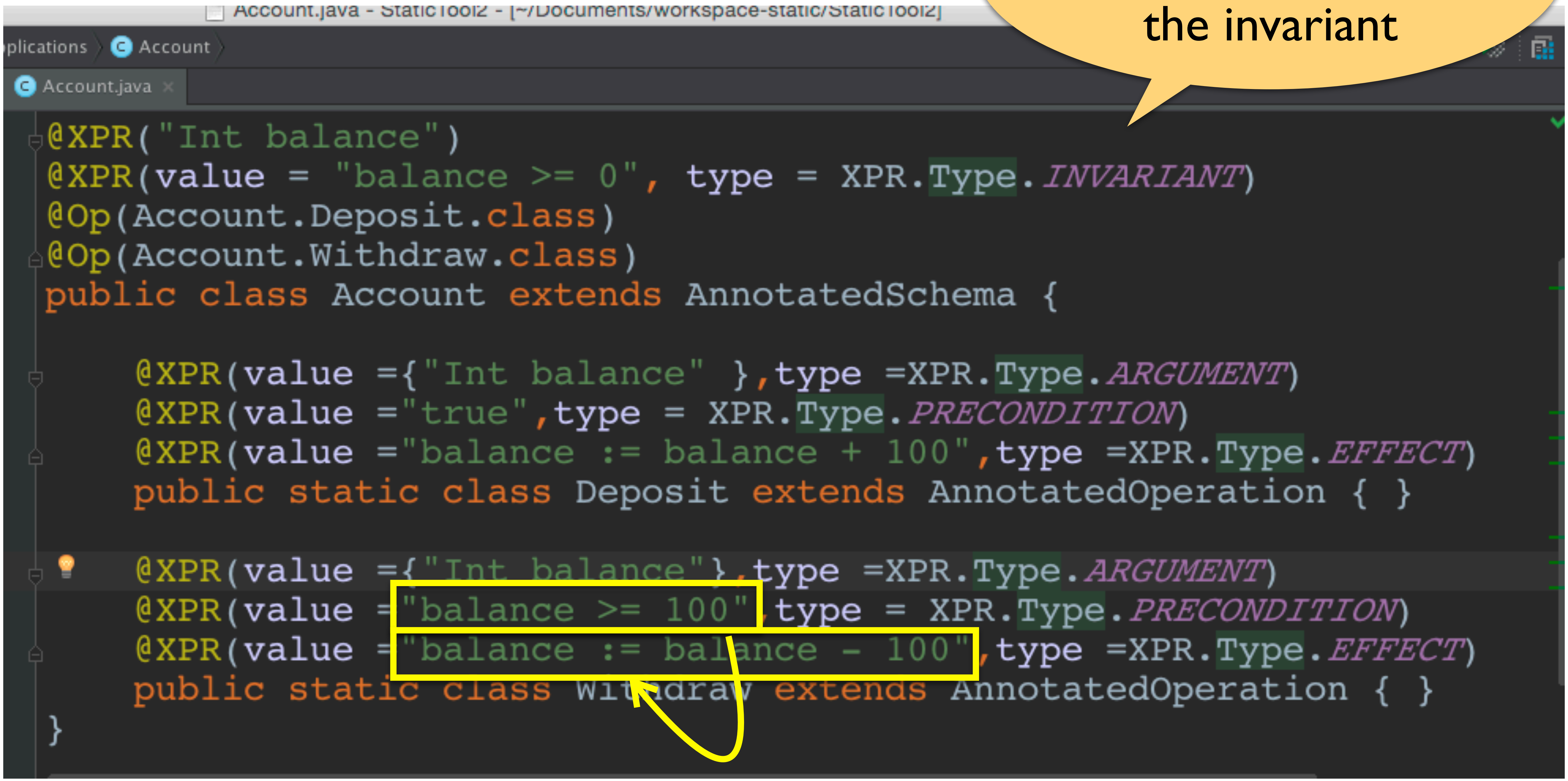
2. Precondition stability: P is preserved by concurrent operations

Bug: concurrent withdrawals may violate the invariant



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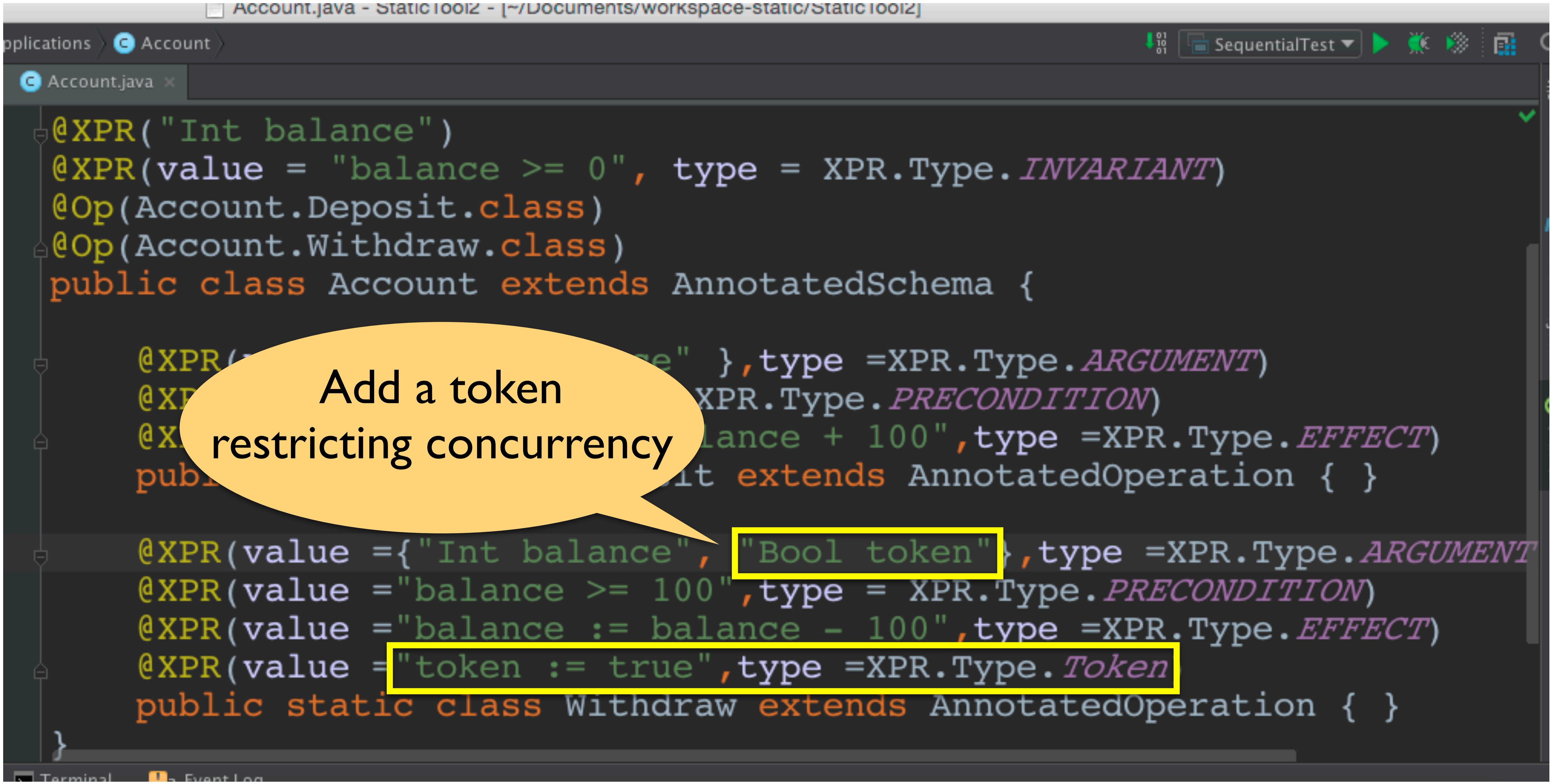

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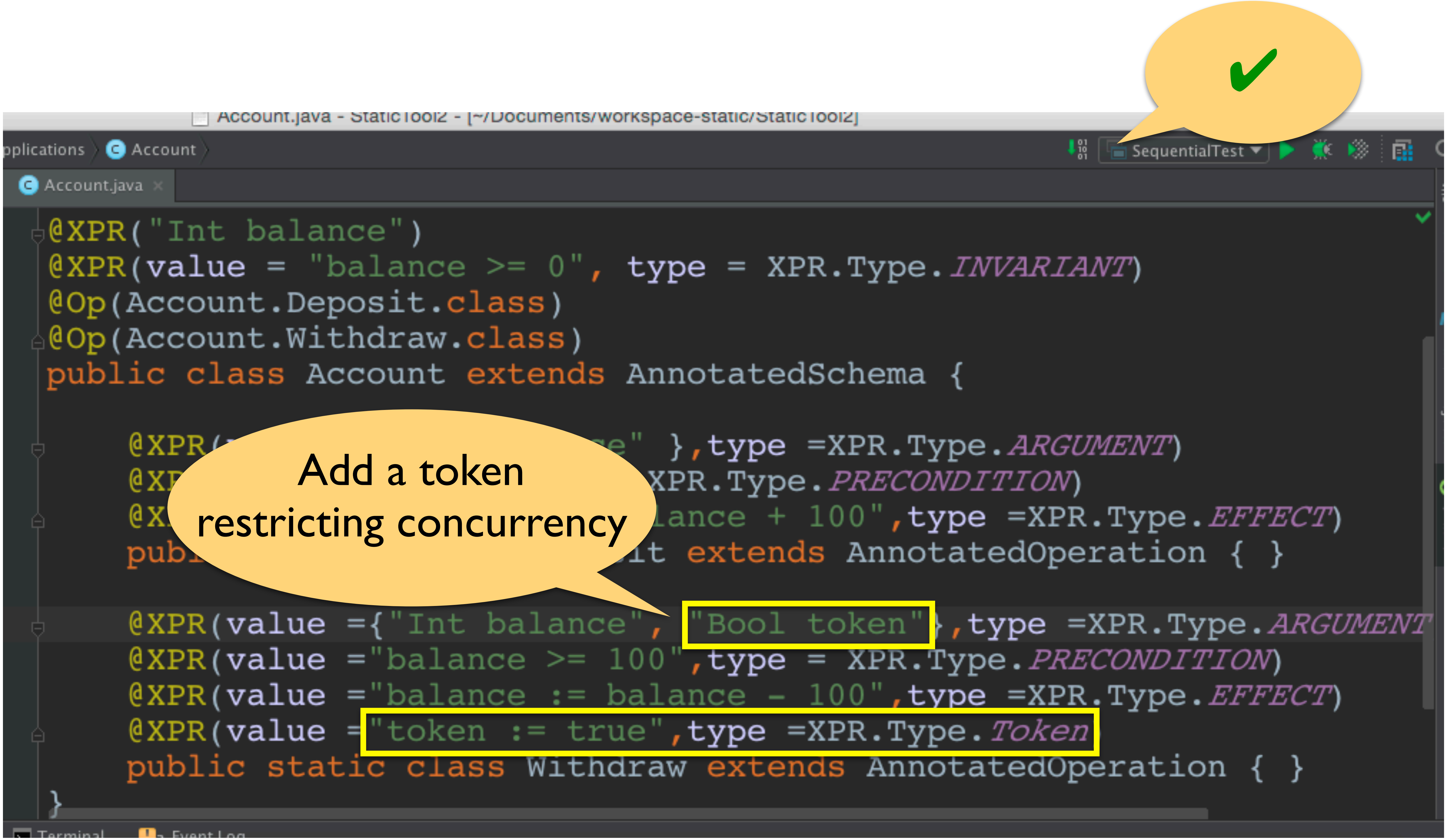
Add a token restricting concurrency

2. Precondition stability: P is preserved by concurrent operations



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Add a token restricting concurrency

Conclusion

- First proof rule and tool for proving invariants of weakly consistent applications
- Case studies: fragments of web applications, replicated file system in progress
- Future work: other consistency models, automatic inference of consistency levels